8/5/2020

Aminone Oghenevwakpo Alexander MAT106

19/ENG05/010 Mechatronics Engineering

1) $$\frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{A}{(x-1)} + \frac{B}{(x-2)} + \frac{C}{(x-3)}$$

Using LCM

$$3x-1 = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)$$

when $x = 1$

$3(1)-1 = A(-1)(-2)$

$3-1 = +2A$

$+2 = +2A$

$A = +2/2$

$A = 1$

when $x = 2$

$3(2)-1 = B(1)(-1)$

$6-1 = -B$

$5 = -B$

$B = -5$

when $x = 3$

$3(3)-1 = C(2)(1)$

$9-1 = 2C$

$8 = 2C$

$C = 8/2 = 4$

$$\therefore \int \frac{3x-1}{(x-1)(x-2)(x-3)} = \int \frac{1}{(x-1)}\,dx + \int \frac{-5}{(x-2)} + \int \frac{4}{(x-3)}$$

$$= \ln(x-1) - 5\ln(x-2) + 4\ln(x-3) + C$$

19/ENG05/010

1) $\int \frac{x^2+x+1}{(x+2)(x^2+1)}dx = \frac{A}{x+2} + \frac{Bx+C}{x^2+1}$

using Lcm

$x^2+x+1 = A(x^2+1) + (Bx+C)(x+2)$

when $x = -2$

$(-2)^2 - 2 + 1 = A(4+1)$

$4 - 2 + 1 = 5A$

$3 = 5A$

$A = 3/5$

Expand the brackets above

$x^2+x+1 = Ax^2 + A + Bx^2 + 2Bx + Cx + 2C$

Compare both sides

$1 = A + B$ ... (i)

$1 = 2B + C$ ... (ii)

$1 = A + 2C$ ... (iii)

In equ (i)

$1 = \frac{3}{5} + B$

$B = 1 - 3/5 = 2/5$

In equ (iii)

$1 = 3/5 + 2C$

$2C = 1 - \frac{3}{5}$

$2C = 2/5$

$C = 2/5 \times \frac{1}{2}$

$C = 1/5$

19/ENG05/010

1 ⇒ $\int \frac{x^2+x+}{(x+2)(x^2+1)}dx = \int \frac{3}{5(x+2)}dx + \int \frac{2x+1}{5(x^2+1)}dx$

Ans = $\frac{3}{5}\ln(x+2) + \frac{1}{5}\ln(x^2+1) + \underline{\frac{1}{5}\tan^{-1}(x) + C}$

How did i get the underlined part?

$\frac{1}{5}\int \frac{2x+1}{x^2+1}dx = \frac{1}{5}\left(\int \frac{2x}{x^2+1}dx + \frac{1}{x^2+1}dx\right)$

$= \frac{1}{5}\left(\ln(x^2+1) + \arctan x\right) + C$

$= \frac{1}{5}\ln(x^2+1) + \frac{1}{5}\tan^{-1}x + C$

$\therefore = \frac{3}{5}\ln(x+2) + \frac{1}{5}\ln(x^2+1) + \frac{1}{5}\tan^{-1}x + C$

3) $\frac{x^2+1}{(x-3)(x-2)^2} = \frac{A}{x-3} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$

using LCM

$x^2+1 = A(x-2)^2 + B(x-3)(x-2) + C(x-3)$

when $x = 3$

$3^2+1 = A(3-2)^2$

$10 = A$

14/ENG05/010

Expanding the bracket above

$x^2+1 = A(x^2-4x+4) + B(x^2-5x+6) + C(x-3)$

$x^2+1 = Ax^2 - 4Ax + 4A + Bx^2 - 5Bx + 6B + Cx$
$- 3C$

comparing both equations

$1 = A + B \quad \text{---} (1)$

$1 = +4A + 6B - 3C \quad \text{--} (11)$

In equ (1)

$1 = 10 + B$

$B = -9$

In equ (11)

$1 = 4(10) + 6(-9) - 3C$

$1 = 40 - 54 - 3C$

$3C = -1 + 40 - 54$

$3C = -15$

$C = -5$

$\therefore \int \frac{x^2+1}{(x-3)(x-2)^2} dx = \int \frac{10}{(x-3)} dx + \int \frac{-9}{(x-2)} + \int \frac{-5}{(x-2)^2}$

$= 10\ln(x-3) - 9\ln(x-2) + 5(x-2)^{-1} + c$

P

19/ENG05/010

1. (4) D) $\int \frac{x^3+x^2+x+1}{x-1}\,dx$

=

$$\begin{array}{r|l} & x^2+2x+3 \\ x-1 & x^3+x^2+x+1 \\ & \underline{x^3-x^2} \\ & 2x^2+x+1 \\ & \underline{2x^2-2x} \\ & 3x+1 \\ & \underline{3x-3} \\ & 4 \end{array}$$

$\therefore \int \frac{x^3+x^2+x+1}{x-1}\,dx = \int (x^2+2x+3)\,dx + \int \frac{4}{x-1}\,dx$

$= \frac{x^3}{3} + x^2 + 3x + 4\ln(x-1) + C$

13.5