Assignment
System Response 1

Question 1

$k_d = 0.05$

$k = 4\,kN/m$

$F = 100N$

$x = 16mm$

$$G(s) = \frac{x(s)}{F(s)} = \frac{1/k}{\tau s + 1}$$

$$F = kx + k_d\, dx/dt \quad \text{—— (i)}$$

$$F = \frac{B\,dx}{dt} + kx$$

$\mathcal{L}(t)$ of (i)

$$F(s) = kx + k_d s x$$

$$F(s) = x(k + k_d s)$$

$$\frac{F(s)}{x} = k + k_d s$$

Taking inverse

$$\frac{x}{F(s)} = \frac{1}{k + k_d s}$$

$$= \frac{1}{k_d s + k}$$

$$\frac{x}{F(s)} = \frac{1/k}{(k_d/k)s + 1} = \frac{1/k}{\tau s + 1}$$

$$T = \frac{k_d}{k}$$

$$= \frac{0.03}{4 \times 10^3}$$

$$T = 7.5 \times 10^{-6}\,secs$$

x_0 after T secs

$$x_0 = F/k\,(1 - e^{-1})$$

$$= \frac{100}{4 \times 10^3}(1 - e^{-1})$$

$$= \frac{100}{4 \times 10^3} \times 0.632$$

$$= 0.016m$$

$$\simeq 16mm$$

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Question 2

$m = 0.5kg$

$shc = 346$

$\theta_1 = 20°c$

$\theta_2 = 120°c$

$\theta = 119°$

$$\frac{\theta_1}{\theta_2}(s) = \frac{1}{\tau s + 1}$$

show that $\theta = \theta_1 + (\theta_2 - \theta_1)(1 - e^{-t/T})$

$$\frac{\theta}{\theta_2} = \frac{1}{\tau s + 1}$$

$$\theta = \tau s \theta_2 + \theta_2$$

$$\theta = \theta_2(\tau s + 1)$$

$$\theta(s) = \frac{\theta_2}{\tau s + 1}$$

$$\theta(s) = \frac{1}{s(\tau s + 1)}$$

$$\theta(t) = \frac{1/T}{s(s + 1/T)}$$

$$\theta(t) = 1 - e^{-t/T}$$

$$\theta(t) = \theta_1 + (\theta_2 - \theta_1)(1 - e^{-t/T})$$

$$119 = 20 + (120 - 20)(1 - e^{-6/T})$$

$$99 = 100(1 - e^{-6/T})$$

$$0.99 = (1 - e^{-6/T})$$

$$0.99 - 1 = -e^{-6/T}$$

$$-0.01 = -e^{-6/T}$$

$$\ln 0.01 = -6/T$$

$$-4.605 = -6/T$$

$$T = 6/4.605$$

$$T = 1.302 \text{ mins}$$

$$= 78.17 \text{ secs.}$$

$\therefore$ thermal capacitance $C = mc$

$= 0.5 \times 346 = 173 J/k$

$T = RC$

$R = T/C$

$R = 78.17/173$

$\therefore R = 0.451 \, k/W$

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Question 3

$$\frac{\omega}{K_m x} = \frac{1}{\tau s + 1}$$

$$T = \frac{J}{k_3}, \quad k_m = \frac{k_1 k_2}{k_3}$$

$k_1, k_2, \omega, J, k_3, k_m$

Assuming no load torque

Torque $= k_p$

$= Ja + k_3\omega$

$k_1 k_2 x(s) = Js(\omega) + k_3\omega$

$(k_1 k_2 / k_3) x(s) = \omega\{(J/k_3)s + 1\}$

$$\frac{\omega}{K_m x} = \frac{1}{Ts + 1}$$

$k_1 k_2 x = J d\omega/dt + k_2\omega$

$k_1 k_2 x(s) = \omega(Js + k_3)$

$K_m x(s) = \omega\{Ts + 1\}$

$$\frac{\omega}{K_m x} = \frac{x}{\tau s + 1}$$

substituting $x(s) = 1/s$

$$\frac{\omega}{k_m}(t) = \frac{1}{s(\tau s + 1)}$$

rearranging

$$= \frac{1/T}{s(s + 1/T)}$$

$$\frac{\omega}{k_m}(t) = (1 - e^{-t/T})$$

sub $t = T$ $\omega(t) =$

$k_m(1 - e^{-1}) = 0.632 k_m$

$= 63.2\%$

sub $t = 4T$ $\omega(t) =$

$k_m(1 - e^{-4}) = 0.982 k_m$

$= 98.2\%$

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Question 4

$$\frac{\theta_o}{\theta_i}(s) = \frac{1}{\tau s + 1}$$

$\theta_i(t) = ct$

$\theta_i(s) = c/s^2$

$$\theta_o(s) = \frac{\theta_i(s)}{\tau s + 1}$$

$$= \frac{c/s^2}{\tau s + 1}$$

$$= \frac{c}{s^2(\tau s + 1)}$$

$\theta_o = c\{t - \tau(1 - e^{-t/\tau})\}$

$\theta_e(t) = c\tau$

$G(s) = 1/(3s+1)$, $c = 4$, $t = 2$

After 2sec

$\theta_i = 4 \times 2 = 8mm$

$\theta_o = 4(2 - 3)(1 - e^{-2/3}))$

$= 4(2 - 3(0.486))$

$= 4(2 - 1.459)$

$= 4(0.540)$

$= 2.161 mm$

$\theta_e = 8 - 2.161$

$= 5.838$

steady state error is $c\tau$

$= 4 \times 3$

$= 12mm$

Question 5

(i) $$G(s) = \frac{2}{0.2s + 0.5}$$

taking $s = 0$

$$\frac{2}{0.2 \times 0 + 0.5}$$

$$= \frac{2}{0.5}$$

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DC gain = 4

Time constant = $\frac{2/0.5}{(0.2/0.5)s+1}$

$= \frac{4}{0.4s+1}$

$= 0.4$

(ii) $G(s) = \frac{0.2}{0.05s + 0.1}$

At $s = 0$

DC gain = $\frac{0.2}{0.1} = 2$

time constant = $\frac{0.05}{0.1} = 0.5$

(iii) $G(s) = \frac{2}{3s+1}$

DC gain = 2

Time constant = 3

(iv) $G(s) = \frac{16}{8s+4}$

DC gain = 4

Time constant = 8/4

= 2

Question 6

$\frac{\omega}{\theta}(s) = \frac{K_m}{T_m s + 2}$

where $K_m = 15s^{-1}$ and $T_m = 4s$

$\frac{\omega}{\theta}(s) = \frac{15}{4s+2}$

DC gain = $7.5s^{-1}$

Time constant = 4/2

= 2 secs

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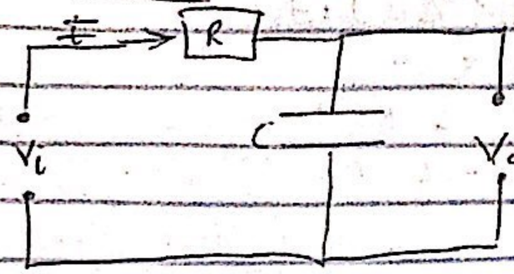
System Response 2

Question 1



$$\frac{V_o(s)}{V_i} = \frac{1}{\tau s + 1} \quad \text{where } T = RC$$

Given $R = 47\Omega$

$C = 20\mu F$

$V_i = 5\sin(2000t)$

$V_i = IR + I/Cs$

$V_i = I(R + 1/Cs)$

$V_o = I/Cs$

$$G(s) = \frac{V_o}{V_i}(s) = \frac{1/Cs}{I(R + 1/Cs)}$$

$$= \frac{1}{RCs + 1}$$

$T = RC$

$$G(s) = \frac{1}{\tau s + 1}$$

$T = 47 \times (20 \times 10^{-6})$

$= 940 \times 10^{-6}\, s$

$\phi = -\tan^{-1}(\omega T)$

$= -\tan^{-1}(2000 \times 940 \times 10^{-6})$

$= -62°$

$$\frac{\theta_o}{\theta_1} = \frac{1}{\sqrt{1 + T^2\omega^2}}$$

$$= \frac{1}{\sqrt{1 + (940 \times 10^{-6})^2 \times (2000)^2}} = 0.47$$

$\therefore \theta_o = (5 \times 0.47)\sin(2000t - 62°)$

$= 2.35\sin(2000t - 62°)$

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Question 2

$$\frac{X_o}{X_i} = \frac{1}{(T^2s^2 + 2\delta Ts + 1)}$$

$T = 0.4$ secs

$\delta = 0.2$

$\theta_i = 6\sin(\omega t)$ @ 2.5 rad/s

Taking C and D as constants

$$C = \frac{(1 - \omega^2 T^2)}{\{(1-\omega^2T^2) + (2\delta\omega T)^2\}}$$

$$C = \frac{(1 - (2.5^2)(0.4^2))}{\{(1 - 2.5^2 \cdot 0.4^2) + (2 \times 0.2 \times 2.5 \times (0.4))^2\}}$$

$$C = \frac{0}{\{0 + (0.4)^2\}}$$

$$C = 0$$

$$D = \frac{2\delta T\omega}{\{(1 - T^2\omega^2)^2 + (2\delta T\omega)^2\}}$$

$$= \frac{2 \times 0.2 \times 0.4 \times 2.5}{\{(1 - 0.4^2 \cdot 2.5^2)^2 + (2 \times 0.2 \times 2.5 \times 0.4)^2\}}$$

$$= \frac{0.4}{\{0 + (0.4)^2\}}$$

$$= \frac{0.4}{0.16}$$

$$= 2.5$$

$$\phi = -\tan^{-1}(D/C)$$

$$= -\tan^{-1}(\infty)$$

$$\therefore \theta = 90^\circ$$

$$\left|\frac{\theta_o}{\theta_i}\right| = \sqrt{D^2 + C^2}$$

$$= \sqrt{2.5^2 + 0^2}$$

$$= 2.5$$

$$\theta = 6 \times 2.5$$

$$\theta = 15^\circ$$

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