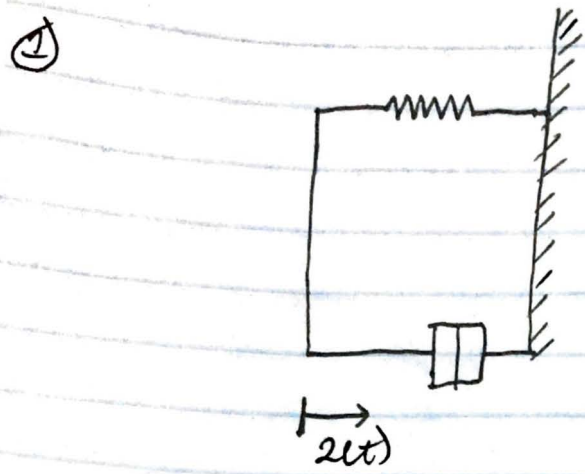


17/2004/053  
 Elect. Syst.  
 Linear Systems.



Spring  $\rightarrow k(x-\theta)$   
 2 (damper)  $\rightarrow kd \frac{d(x-\theta)}{dt}$   
 $2(x) \rightarrow 2(x)$

From Newton's law,  $\therefore 2(x) - k(x-\theta) - kd \frac{d(x-\theta)}{dt} = 0$

$\rightarrow 0 = 2(x) - kx - kd \frac{dx}{dt}$

Laplace:  $\mathcal{L}(2(x) - kx(s) - kd s x(s)) = 0$   
 $\Rightarrow 2(x(s) - kx(s) - kd s x(s)) = 0$

$2(x(s) = [k + kd s] x(s)$   
 $G(s) = \frac{X(s)}{2(x(s)} = \frac{1}{k + kd s}$

$= \frac{1/k}{1 + [kd/k] s}$

From  $\frac{1/R}{Ts+1}$

$T = kd/k = \frac{0.05}{4 \times 10^3}$   
 $0.75 \times 10^{-5}$

$$\varepsilon_2 \approx mc \theta_2 \approx mc [\theta_2 - \theta_1]$$

$$\varepsilon_1 \approx mc [\theta_2 - \theta_1]$$

$$G(s) = \frac{\varepsilon_2}{\varepsilon_1} = \frac{mc [\theta_2 - \theta_1]}{mc [\theta_2 - \theta_1]}$$

$$= \frac{1}{Ts+1}$$

$$\frac{\theta - \theta_1}{\theta_2 - \theta_1} = \frac{1}{Ts+1}$$

$$\theta - \theta_1 = \frac{\theta_2 - \theta_1}{Ts+1}$$

$$\text{Let } \theta_2 - \theta_1 = K(t)$$

$$\rightarrow (\theta_2 - \theta_1)(s) = \frac{K(s)}{Ts+1}$$

$$\Rightarrow (\theta_2 - \theta_1)(s) = \frac{K(s) \left(\frac{1}{s}\right)}{s + \frac{1}{T}}$$

Laplace of  $\theta_2 - \theta_1$  is  $K(s)$

$$\rightarrow (\theta_2 - \theta_1)(s) = \frac{K \left(\frac{1}{s}\right)}{s \left(s + \frac{1}{T}\right)}$$

$$\mathcal{L}^{-1}(\theta_2 - \theta_1) = K(1 - e^{-t/T})$$

(3)

$$\frac{W}{K_m \tau} = \frac{1}{Ts+1}$$

$$\tau = \tau_{KS}$$

$$K_m = \frac{K_1 K_2}{K_S}$$

$$W = \frac{K_m \tau}{Ts+1}$$

Laplace of step input

$$W = \frac{K_m \tau}{s} \left(\frac{1}{Ts+1}\right)$$

$$\frac{K_m \tau}{s} \left(\frac{1}{s + \frac{1}{T}}\right)$$

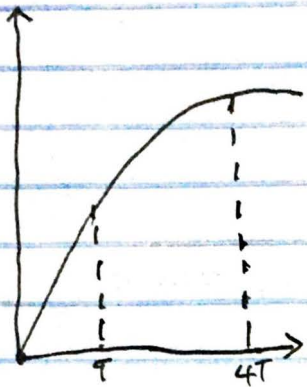
$$W(s) = K_m \tau (1 - e^{-t/T})$$

$t=0$ ;  $k_{mn}(1-e^0) = \text{Horizontal}$   
 $t=T$ ,  $k_{mn}(1-e^{-T/T}) = 0.632 k_{mn}$   
 $t=4T$ ,  $k_{mn}(1-e^{-4}) = 0.981 k_{mn}$   
 For  $t=T$

$\Delta\% = (0.632) \times 100\% = 63.2\%$

$t=4T$

$\Delta\% = (0.981 - 0) \times 100\% = 98.1\%$



$\Theta(t) = Ct$

$\Theta_1(t) = e^{-Ct/s}$

$\frac{\Theta_2(s)}{\Theta_1} = \frac{1}{s^2 + 1}$

$\Theta_0(s) = \frac{\Theta_1}{s^2 + 1}$

$\rightarrow \Theta_0(s) = \frac{C}{s^2} \div (s^2 + 1)$

$\frac{C}{s^2(s^2 + 1)}$

$\Theta_0 = \frac{1/3}{s^2(s^2 + 1/3)}$

$\Theta_0(t) = [Lt - 3(1 - e^{-t/3})] - \text{①}$

(a)  $t$  is close

$\Theta_0(t) = (t - 3C)$

$\Theta_0(t) = (t - 3C)$

$\Theta_1 = \Theta_0 - \Theta_0 = (t - [t - 3C]) = (t - t + 3C) = 3C$

$= 3C$  After 2 sec

$\Theta_1 = 4 \times 2 = 8 \text{ mm}$

$\Theta_2 = 4 \times 3 = 12 \text{ mm}$

$\therefore \Theta_0(t) = C [t - 3(1 - e^{-t/3})]$

$\uparrow = 3, C = 4 \text{ mm/s}$

$$\frac{2}{0.25s+0.5} = \frac{2/0.5}{\frac{0.25}{0.5}s+1}$$

$$= \frac{4}{0.5s+1} \quad \text{from } \frac{k}{Ts+1}$$

$\therefore$  4 = DC gain, 0.5 = time constant.

$$\textcircled{ii} \quad \frac{0.2}{0.05s+0.1} = \frac{0.2/0.1}{\frac{0.05}{0.1}s+\frac{0.1}{0.1}}$$

$$= \frac{2}{0.5s+1} \quad \text{from } \frac{k}{Ts+1}$$

$\Rightarrow$  2 = DC gain, 0.5 = time constant

$$\textcircled{iii} \quad \frac{2}{3s+1} \Rightarrow \text{from } \frac{k}{Ts+1}$$

$\Rightarrow$  2 = DC gain, 3 = time constant

$$\textcircled{iv} \quad \frac{16}{8s+4} = \frac{16/4}{\frac{8}{4}s+\frac{4}{4}}$$

$$\frac{4}{2s+1} \quad \text{from } \frac{k}{Ts+1}$$

$\Rightarrow$  4 = DC gain, 2 = time constant.

$\textcircled{b}$

$$\frac{w(s)}{\theta} = \frac{km}{Tms+2}$$

$$km = 1.5^{s+1}$$

$$Tm = 4$$

$$\Rightarrow \frac{15}{4s+2} = \frac{15/2}{4^{s/2}+1} = \frac{7.5}{2s+1}$$

$$\frac{w(s)}{\theta} = \frac{km}{Tms+2}$$

$$km = 1.5^{s+1}$$

$$Tm = 4$$

$$\frac{1}{4s+2} = \frac{15/2}{4^{s/2}+1}$$

$\therefore$  7.5 = DC gain, 2 = time constant.