

$$1. \int \frac{(3x-1)}{(x-1)(x-2)(x-3)} dx$$

$$2. \int \frac{(x^2+x+1)}{(x+2)(x^2+1)} dx$$

$$3. \int \frac{(x^2+1)}{(x-3)(x-2)^2} dx$$

$$4. \int \frac{(x^3+x^2+x+1)}{(x-1)} dx$$

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Assignment

Find the integral of the following

$$1) \int \frac{(3x-1)}{(x-1)(x-2)(x-3)} dx$$

Solution

$$\int \frac{3x-1}{(x-1)(x-2)(x-3)} =$$

$$\frac{(3x-1)}{(x-1)(x-2)(x-3)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$$

$$\frac{A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)}{(x-1)(x-2)(x-3)}$$

$$= 3x-1 = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)$$

when $x=2$

$$3(2)-1 = A(2-2)(2-3) + B(2-1)(2-3) + C(2-1)(2-2)$$

$$6-1 = -B$$

$$B = -6+1$$

$$\frac{B}{1} = \frac{-5}{+1}$$

$$B = -5$$

when $x=3$

$$3(3)-1 = A(3-2)(3-3) + B(3-1)(3-3) + C(3-1)(3-2)$$

$$9-1 = C(3-1)(3-2)$$

$$8 = 2C$$

$$\frac{2C}{2} = \frac{8}{2}, C = 4$$

when $x = 1$

$$3(1-1) = A(1-2)(1-1) + B(1-1)(1-1) + C(1-1)(1-2)$$

$$3-1 = A(1-2)(1-1)$$

$$2 = A(-1)(-2)$$

$$2 = 2A$$

$$A = 1$$

$$\int \frac{(3x-1)}{(x-1)(x-2)(x-3)} = \int \frac{1}{x-1} + \frac{-5}{x-2} + \frac{4}{x-3}$$

$$= \ln|x-1| - 5\ln|x-2| + 4\ln|x-3|$$

$$2 \frac{(x^2+x+1)}{(x+2)(x^2+1)} dx = \frac{x^2+x+1}{x^5+x+2x^2+2}$$

$$= \frac{x^2+x+1}{x^5+2x^2+x+2}$$

$$= \frac{Ax+B}{x^2+1} + \frac{C}{x+2} = \frac{x^2+x+1}{(x^2+1)(x+2)}$$

$$\frac{(Ax+B)(x+2) + C(x^2+1)}{(x^2+1)(x+2)} = \frac{x^2+x+1}{(x^2+1)(x+2)}$$

$$(Ax+B)(x+2) + C(x^2+1) = x^2+x+1$$

$$Ax^2 + 2Ax + Bx + 2B + Cx^2 + C = x^2 + x + 1$$

$$(A+C)x^2 + x(2A+B) + (2B+C) = x^2 + x + 1$$

$$A+C = 1 \quad \text{--- (1)}, \quad A = 1-C$$

$$2A+B = 1 \quad \text{--- (2)}$$

$$2B+C = 1 \quad \text{--- (3)}$$

Put $A = 1-C$ in equation (2)

$$2C(1-C) + B = 1$$

$$2 - 2C + B = 1$$

$$-2C + B = 1 - 2$$

$$-2C + B = -1$$

$$B - 2C = -1 \quad \text{--- (4)}$$

$$2B + C = 1 \quad \text{--- (5)}$$

Subtract equation (4) from (3)

$$2B + C = 1$$

$$B - 2C = -1 \times 2$$

$$= 2B - 4C = -2$$

$$\underline{2B + C = 1}$$

$$-5C = -3$$

$$C = \frac{-3}{-5}$$

$$C = \frac{3}{5}$$

Put $C = 3/5$ into equation (1)

$$A + C = 1$$

$$A + \frac{3}{5} = 1$$

$$A = 1 - 3/5$$

$$A = 2/5$$

Put $C = 3/5$ into equation (3)

$$2B + 3/5 = 1$$

$$2B = 1 - 3/5$$

$$2B = 2/5$$

$$B = 2/5 \times 1/2$$

$$B = 1/5$$

$$\int \frac{x^2 + x + 1}{(x^2 + 1)(x + 2)} dx = \int \frac{x^2 + x + 1}{x^2 + 1} + \int \frac{dx}{x + 2}$$

$$3. \int \frac{(x^2+1)}{(x-3)(x-2)^2} dx.$$

Solution

$$\frac{A}{x-3} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$$

$$\frac{A(x-2)^2 + B(x-3)(x-2) + C(x-3)}{(x-3)(x-2)^2}$$

$$= A(x-2)^2 + B(x-3)(x-2) + C(x-3) = x^2 + 1$$

let $x = 2$

$$2^2 + 1 = A(2-2)^2 + B(2-3)(2-2) + C(2-3)$$

$$4 + 1 = C(-1)$$

$$5 = -C$$

$$C = -5$$

let $x = 3$

$$3^2 + 1 = A(3-2)^2 + B(3-3)(3-2) + C(3-3)$$

$$10 = A(1)^2$$

$$A = 10$$

Comparing of coefficient of x^2 ; $A + B = 1$

$$10 + B = 1$$

$$B = 1 - 10$$

$$B = -9$$

$$\frac{10}{x-3} + \frac{-9}{x-2} + \frac{-5}{(x-2)^2}$$

$$= \frac{10}{x-3} - \frac{9}{x-2} - \frac{5}{(x-2)^2}$$

$$\int \frac{x^2+1}{(x-3)(x-2)^2} dx = \int \frac{10}{x-3} - \int \frac{9}{x-2} - \int \frac{5}{(x-2)^2}$$

$$= 10 \ln|x-3| - 9 \ln|x-2| - \frac{5}{x-2} + C$$

$$= 10 \ln|x-3| - 9 \ln|x-2| - 5 \ln|x-2| + C$$

4. $(x^3 + x^2 + x + 1) / (x - 1)$ dku.

Solution $\frac{x^3 + x^2 + x + 1}{x - 1}$ dku