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Matric no: 19/ENG05/035
Course code: MAT 102

Assignment answers

1.) $M = pi - 6j - 3k$, $N = 4i + 3j - k$, $O = i - 3j + 2k$

a) NOTE: Condition for perpendicularity is that dot product must equal '0'

$\therefore M \cdot N = 0$

$\therefore (4 \times p) + (3 \times -6) + (-1 \times -3) = 0$

~~$4p - 18 + 4 = 0$~~ $4p - 18 + 3 = 0$

$\therefore 4p = 15$

$\therefore p = 15/4$ //

b) NOTE: Condition for co-planarity is that scalar triple product must equal zero

$\therefore m \cdot (N \times O) = 0$

$$N \times O = \begin{vmatrix} i & j & k \\ 4 & 3 & -1 \\ 1 & -3 & 2 \end{vmatrix}$$

$= (3 \times 2) - (-3 \times -1)i - ((4 \times 2) - (1 \times -1))j + ((4 \times -3) + (3 \times 1))k$

$= (6 - 3)i - (8 - (-1))j + (-12 + 3)k$

$= 3i - 9j - 9k$

$m \cdot (N \times O) = (pi - 6j - 3k) \cdot (3i - 9j - 9k)$

$= (3 \times p) + (-6 \times -9) + (-3 \times -9)$

$= 3p + (54) + (27)$

$= 3p - 54 - 27$

$= 3p - 81$

but $m \cdot (N \times O) = 0$

$\therefore 3p - 81 = 0$

$\therefore 3p = 81$

$\therefore p = 81/3 = 27$ //

2) Sum = $3i + 2j + 5k$

$2i - j + 6k$

$5i + 2j - 3k$

$10i + 3j + 8k$ //

Let $x = 10$

$y = 3$

$z = 8$

$\cos a = \frac{x}{\text{length}}$

$\text{length} = \sqrt{10^2 + 3^2 + 8^2} = \sqrt{100 + 9 + 64} = \sqrt{173}$

$\therefore \cos a = \frac{10}{\sqrt{173}}$

$\cos b = \frac{3}{\sqrt{173}}$

$\cos c = \frac{8}{\sqrt{173}}$

$\therefore$ Direction cosine $= \left(\frac{10\sqrt{173}}{173}, \frac{3\sqrt{173}}{173}, \frac{8\sqrt{173}}{173}\right)$

Direction cosines and unit vectors have the same coordinates

$\therefore$ unit vector $= \frac{10\sqrt{173}}{173} i + \frac{3\sqrt{173}}{173} j + \frac{8\sqrt{173}}{173} K$ //

3) $F = (3u)i + (u^2)j + (u+2)k$

$V = (2u)i + (3u)j + (u-2)k$

$\int_0^1 (F \times V)\,du = ?$

Soln

$$(F \times V) = \begin{vmatrix} i & j & k \\ 3u & u^2 & u+2 \\ 2u & 3u & u-2 \end{vmatrix}$$

$(F \times V) = |(u-2)\times u^2) - ((u+2)\times 3u)|i - |((u-2)\times 3u) - ((u+2)\times 2u)|j + |(u^3 \times 2u) - (3u \times 3u)|$

k

$= |u^3 - 2u^2|i$

$= ((u^3 - 2u^2) - (3u^2 + 6u))i - ((3u^2 - 6u) - (2u^2 + 4u))j + (2u^3 - 9u^2)k$

$(F \times V) = (u^3 - 6u^2 - 6u)i - (u^2 - 10u)j + (2u^3 - 9u^2)k$ //

$$\int_0^1 (F \times V)\,du = \left(\frac{u^4}{4} - \frac{6u^3}{3} - \frac{6u^2}{2}\right)i - \left(\frac{u^3}{3} - \frac{10u^2}{2}\right)j + \left(\frac{2u^4}{4} - \frac{9u^3}{3}\right)k \Bigg|_0^1$$

$$= \left(\frac{u^4}{4} - 2u^3 - 3u^2\right)i - \left(\frac{u^3}{3} - 5u^2\right)j + \left(\frac{u^4}{2} - 3u^3\right)k \Bigg|_0^1$$

$$= \left(\frac{(1)^4}{4} - 2(1)^3 - 3(1)^2\right)i - \left(\frac{(1)^3}{3} - 5(1)^2\right)j + \left(\frac{(1)^4}{2} - 3(1)^3\right)k - 0$$

$$= \left(\frac{1}{4} - 2 - 3\right)i - \left(\frac{1}{3} - 5\right)j + \left(\frac{1}{2} - 3\right)k$$

$$= \left(\frac{-19}{4}\right)i - \left(\frac{-14}{3}\right)j + \left(\frac{-5}{2}\right)k$$

$$= -\frac{19}{4}i + \frac{14}{3}j - \frac{5}{2}k$$