EZEOBIDI CLEMENTINA ONYINYECHUKWU

17/ENG04/027

⇒ <u>Solutions to System response II</u>

Q1) given : $R = 47\Omega$, $C = 20\mu F = 20\times10^{-6} f$

$V_i = 5\sin(2000t)$

$$\frac{V_o}{V_i}(s) = \frac{1}{Ts+1} \quad \text{but } T = RC$$

<u>Soln</u>

$$\frac{V_o}{V_i}(j\omega) = \frac{1}{1+Tj\omega} = \frac{(1-Tj\omega)}{(1+Tj\omega)(1-Tj\omega)}$$

$$\frac{V_o}{V_i}(j\omega) = \frac{1-Tj\omega}{1+T^2\omega^2} = \frac{1}{1+T^2\omega^2} - \frac{jT\omega}{1+T^2\omega^2}$$

recall : Phase angle, $\phi = \tan^{-1}\left(\frac{B}{A}\right)$

but $B = \frac{T\omega}{1+T^2\omega^2}$

⇒ recall! $T = RC = 47\times20\times10^{-6} = 9.4\times10^{-4}$ sec

$$\therefore B = \frac{9.4\times10^{-4}\times2000}{1+(9.4\times10^{-4})^2\times(2000)^2} = 0.4146$$

$$A = \frac{1}{1+T^2\omega^2}$$

$$A = \frac{1}{1+(9.4\times10^{-4})^2\times(2000)^2} = 0.22053$$

$$\phi = \tan^{-1}\left(\frac{0.4146}{0.22053}\right) = -61.990^\circ \simeq -62^\circ$$

The output voltage is given by

$$\text{Mod } \frac{\theta_o}{\theta_i} = \sqrt{B^2 + A^2} = \sqrt{(0.4146)^2 + (0.22053)^2}$$

$$\text{mod } \frac{\theta_o}{\theta_i} = 0.4696$$

Hence : output Voltage $= 0.4696 \times 5$

$$= 2.348$$

Therefore : $2.348 \sin(2000t - 62^\circ)$

Q2) given: $T = 0.4\,sec$, damping ratio, $\delta = 0.2$

$\theta_i = 6\sin(wt)$, $w = 2.5\,rad/s$

soln

$$\frac{X_o}{X_i}(s) = \frac{1}{T^2S^2 + 2\delta TS + 1}$$

$$\frac{X_o}{X_i}(jw) = \frac{1}{T^2(jw)^2 + 2T\delta jw + 1} = \frac{1}{(1 - T^2w^2) + 2\delta Tjw}$$

where : $A = 1 - T^2 w^2$

$B = 2\delta T w$

Similar to $\frac{\cancel{X_o}}{\cancel{X_i}} = A + jB$

$$\frac{X_o}{X_i} = \frac{1}{(A+jB)} \times \frac{A-jB}{(A-jB)} = \frac{A-jB}{A^2+B^2}$$

$$\frac{X_o}{X_i} = \frac{A}{(A+B)^2} - j\frac{B}{(A+B)^2} = C - jD$$

$$\therefore C = \frac{1-T^2\omega^2}{(1-T^2\omega^2)^2 + (2\delta T\omega)^2}$$

$$D = \frac{2\delta T\omega}{(1-T^2\omega^2)^2 + (2\delta T\omega)^2}$$

$$\therefore A = 1 - T^2\omega^2 = 1 - (0.4)^2 \times (2.5)^2 = 0$$

$$B = 2\delta\omega T = 2 \times 0.2 \times 2.5 \times 0.4 = 0.4$$

$$\text{but } C = \frac{A}{A^2+B^2} = \frac{0}{0^2+0.4^2} = 0$$

$$D = \frac{B}{A^2+B^2} = \frac{0.4}{0^2+0.4^2} = 2.5$$

Phase angle, $\phi = \tan^{-1}\left(\frac{D}{C}\right)$

$$\phi = \tan^{-1}\left(\frac{2.5}{0}\right) = \tan^{-1}(\infty) = 90^{\circ}$$

Output amplitude:

$$\text{mod}\left(\frac{X_o}{X_i}\right) = \sqrt{2.5^2 + 0^2} = 2.5$$

$\therefore$ output amplitude $= 2.5 \times 6 = 15$

Therefore: $15 \sin(2.5t + 90^{\circ})$

Solutions to System response I

Q1) given that: $K_d = 0.03$

$K = 4 \times 10^3 \ N/m$

recall: $F(t) = Kx$ (spring)

$f(t) = K_d \frac{dx}{dt}$ (damper)

$\Rightarrow$ let: $f(t) = f$

$$\therefore \quad F = Kx = K_d \frac{dx}{dt}$$

Scanned with CamScanner

$$f = K_d \frac{dx}{dt} - kx$$

$$f = K_d\, s - kx$$

$$\frac{f}{x} = \frac{1}{K_d s + k} = \frac{1}{0.03 s + 4\times10^3}$$

Comparing with $\frac{\theta_1}{\theta_2} = \frac{1}{Ts+1}$

$$\frac{f}{x} = \frac{\frac{1}{4\times10^3}}{\frac{0.03}{4\times10^3} s + 1} = \frac{250}{7.5\times10^{-6} s + 1}$$

$\therefore$ The time constant for the system $= 7.5\times10^{-6}$ sec

$= 7.5\ \mu s$

(iii) $X_0 = (F/k)(1-e^{-t/\tau})$
$= (100 \div 4000)(1-e^{-1})$
$X_0 = 0.0158\,m$
$X_0 = \approx 0.016\,m$ OR 16mm

Q2: Given: mass = 0.5 Kg, $c = 346\,J/kg\,K$, $\theta_1 = 20°c$
$\theta_2 = 120°C$, @ $t = 6\,min$ $\theta = 119°c$

take: E_1 = initial energy
E_2 = new energy
but $E_1 = mc\Delta\theta = mc(\theta_2 - \theta_1)$
$E_2 = mc\Delta\theta = mc(\theta - \theta_1)$
NOTE: θ = new metal Temperature

$$G(s) = \frac{E_2}{E_1} = \frac{\cancel{mc}(\theta - \theta_1)}{\cancel{mc}(\theta_2 - \theta_1)} = \frac{1}{Ts+1}$$

$$\therefore \quad \frac{\theta - \theta_1}{\theta_2 - \theta_1} = \frac{1}{Ts+1}$$ (Cross multiply)

$$\theta - \theta_1 = \frac{\theta_2 - \theta_1}{Ts+1}$$

$\Rightarrow$ assuming : $\theta_2 - \theta_1 = K(t)$

$$\therefore \quad \theta - \theta_1 = \frac{K(t)}{Ts+1}$$

$$\theta - \theta_1 = \frac{\frac{K(t)}{T}}{s + \frac{1}{T}}$$

The Laplace Transform of $K(t) = \frac{K}{s}$

$$\theta - \theta_1 = \frac{K(t)(1/T)}{s + T}$$

$$\therefore \theta_2 - \theta_1(s) = \frac{K(1/T)}{s(s+T)}$$

$\Rightarrow$ recalling: similar to exponential growth

$$\theta - \theta_1(t) = K\left[1 - e^{-t/T}\right]$$

recalling : $\theta_2 - \theta_1 = K(t)$

Hence $\theta - \theta_1 = (\theta_2 - \theta_1)\left[1 - e^{-\frac{t}{T}}\right]$

recall: $\theta(t) = \theta_1 + (\theta_2 - \theta_1)(1 - e^{-t/\tau})$

$\therefore$ $119 = 20 + (120 - 20)(1 - e^{-6/T})$

$119 = 20 + 100(1 - e^{-6/T})$

$0.99 = (1 - e^{-6/T})$

$0.99 - 1 = -e^{-6/T}$

$\cancel{-}0.01 = \cancel{-}e^{-6/T}$

$\ln(0.01) = \cancel{\ln}(\cancel{e}^{-6/T})$

$-4.605 = -6/T$

$T = \frac{-6}{-4.605} = 1.304$ minutes

$T = 1.304$ minutes

$T = 78.24$ seconds

Thermal capacitance, $C = mc$

$C = 0.5 \times 346$

$C = 173\ J/K$

$\therefore T = RC \Rightarrow$

$R = \frac{T}{C} = \frac{78.24}{173} = \underline{0.4522\ K/W}$

Scanned with CamScanner

Q3) - $\frac{w}{K_m X} = \frac{1}{Ts+1}$

but $T = \frac{J}{K_3}$, $K_m = \frac{K_1 K_2}{K_3}$

Soln

$$w = \frac{K_m x}{Ts+1}$$

. taking The laplace transform of input step i.e $\frac{1}{s}$

$$W(s) = \frac{K_m x}{s}\left(\frac{1}{Ts+1}\right)$$

$$W(s) = \frac{K_m x}{s}\left(\frac{1/T}{s+1/T}\right) \text{ --- } (*)$$

=> eqn (*) being similar to exponential growth

$$\therefore W(t) = K_m x\left[1-e^{-t/T}\right]$$

=> The % Change in The output at $t=0$

$$= K_m x\left[1-e^{-\frac{0}{T}}\right] = 0$$

at $t = T$

$$= K_m x\left[1-e^{-T/T}\right] = 0.6321 K_m x$$

Scanned with CamScanner

at $t = T$

$= K_m x \left[1 - e^{-\frac{4T}{T}}\right] = 0.98168\, K_m x$

Hence for $t = T$

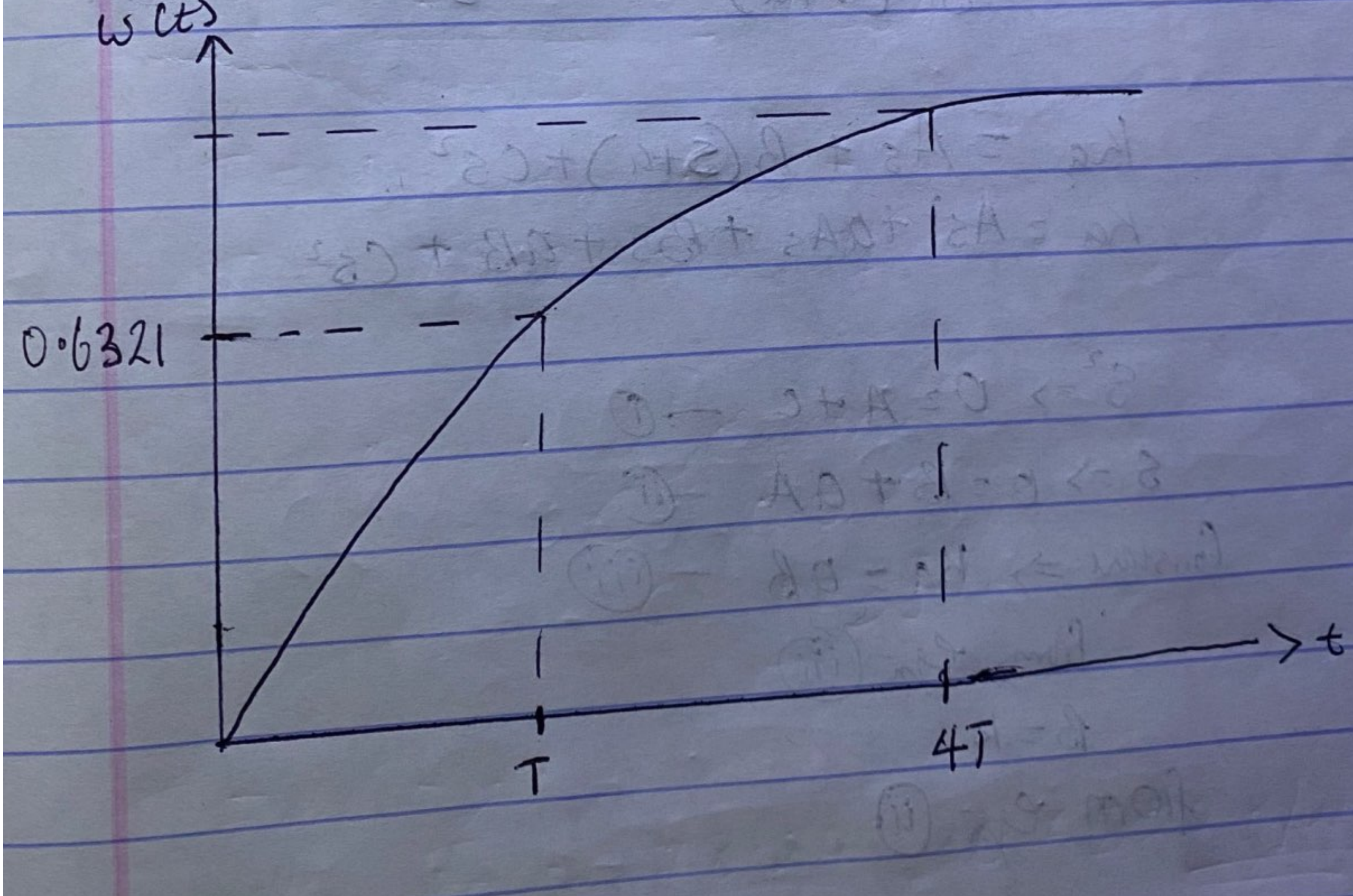
% change in the output $= [0.6321 - 0] \times 100\%$

$= 63.21\% \simeq \underline{63.2\%}$

for $t = 4T$

% change in the output $= [0.98168 - 0] \times 100\%$

$= 98.168\% \simeq \underline{98\%}$



Q4) Given: $G(s) = \frac{1}{3s+1}$

$\Rightarrow M_i \text{ (Input)} = \frac{c}{s^2}$

Soln

$$M_o = \frac{c(1)}{s^2(3s+1)} = \frac{c\left(\frac{1}{3}\right)}{s^2\left(s+\frac{1}{3}\right)}$$

where : $a = 1/3$

$K = c$

$$\therefore \quad M_o = \frac{Ka}{s^2(s+a)} = \frac{As+B}{s^2} + \frac{c}{s+a}$$

$$Ka = As + B(s+a) + Cs^2$$

$$Ka = As^2 + aAs + Bs + aB + Cs^2$$

$s^2 \Rightarrow 0 = A + C$ — (i)

$s \Rightarrow 0 = B + aA$ — (ii)

Constant $\Rightarrow Ka = aB$ — (iii)

from eqn (iii)

$B = K$

from eqn (ii)

$0 = B + aA \Rightarrow 0 = K + aA$

$\frac{-K}{a} = \frac{\cancel{a}A}{\cancel{a}} \Rightarrow A = \frac{-K}{a}$

from eqn (i)

$0 = A + C \Rightarrow 0 = \frac{-K}{a} + C$

$\therefore C = \frac{K}{a}$

$\therefore \frac{\frac{-K}{a}s + K}{s^2} + \frac{\frac{K}{a}}{s+a}$

recall: $a = \frac{1}{3}$

$K = c$

$M_o(s) = \frac{\frac{-K}{a}s + K}{s^2} + \frac{\frac{K}{a}}{s+a}$

$M_o(s) = \frac{\frac{-K}{a}\cancel{s}}{s^{\cancel{2}}} + \frac{K}{s^2} + \frac{\frac{K}{a}}{s+a}$

$M_o(s) = \frac{-K/a}{s} + \frac{K}{s^2} + \frac{\frac{K}{a}}{s+a}$

$M_o(t) = \frac{-c}{\frac{1}{3}} + ct + \frac{c}{\frac{1}{3}}e^{-\frac{1}{3}t} = -3c + ct + 3ce^{-\frac{t}{3}t}$

$$m_o(t) = c\left(-3+t+3e^{-\frac{1}{3}t}\right) = c\left(t-3+3e^{-\frac{1}{3}t}\right)$$

$$m_o(t) = c\left(t-3\left(1-e^{-1/3t}\right)\right)$$

NOTE: at large values of time t, the term $(e^{-t/3})$ becomes negligibly small and hence

$$m_o(t) = c\,(t-3) = ct - 3c$$

the error becomes; $M_e(t) = m_i - m_o$

$$m_e = ct - (ct - 3c) = \cancel{ct} - \cancel{ct} + 3c$$

$$\Rightarrow \quad m_e = 3c$$

recall: $c = 4\,mm/s$; $T = 3$

$\Rightarrow$ after 2 seconds

$t = 2$; $T = 3$, $c = 4\,mm/s$

$$\therefore\ m_i = ct = 4\times10^{-3}\times 2 = 8\times10^{-3}\,m$$

$$m_o = c\left(t-3\left(1-e^{-1/3t}\right)\right)$$

$$m_o = 4\times10^{-3}\left(2-3\left(1-e^{-1/3(2)}\right)\right)$$

$$m_o = 2.161005\times10^{-3}\,m$$

Hence; error, $m_e = m_i - m_o$

$$m_e = 8\times10^{-3} - 2.161005\times10^{-3}$$

$$M_e = 5.839\times10^{-3}\,mm$$

∴ Steady state error, $m_e = 3e$
$$= 3 \times 4 \times 10^{-3} = \underline{\underline{12mm}}$$

Q5) <u>recall</u>: DC gain is the magnitude of $G(s)$ when $s=0$

i) $G(s) = \dfrac{2}{0.2s + 0.5}$

<u>Soln</u>

$$G(s) = \frac{\frac{2}{0.5}}{\frac{0.2}{0.5}s + 1} = \frac{4}{0.4s+1}$$

<u>Hence</u>: DC gain = 4, time Constant = $\underline{\underline{0.4}}$

ii) $G(s) = \dfrac{0.2}{0.05s + 0.1}$

<u>Soln</u>

$$G(s) = \frac{\frac{0.2}{0.1}}{\frac{0.05}{0.1}+1} = \frac{2}{0.5s+1}$$

∴ DC gain = 2, time Constant, $T = \underline{\underline{0.5}}$

iii) $\dfrac{2}{3s+1} = G(s)$

Soln

$$G(s) = \dfrac{\frac{2}{1}}{\frac{3}{1}s+1} = \dfrac{2}{3s+1}$$

∴ Dc gain = 2 , Time Constant, $T = 3$

iv) $G(s) = \dfrac{16}{8s+4}$

Soln

$$G(s) = \dfrac{\frac{16}{4}}{\frac{8}{4}s+1} = \dfrac{4}{2s+1}$$

∴ DC gain = 4 , Time Constant, $T = 2$

Q6: $\dfrac{\omega}{\theta}(s) = \dfrac{K_m}{T_m s+2}$

where: $K_m = 15 s^{-1}$, $T_m = 4 sec$

Soln

∴ $\dfrac{\omega}{\theta}(s) = \dfrac{15}{4s+2}$

Scanned with CamScanner

$$\frac{W}{\theta}(s) = \frac{\frac{15}{2}}{\frac{4}{2}s+1} = \frac{7.5}{2s+1}$$

$\therefore$ D.C gain $= 7.5\,s^{-1}$; Time Constant $= 2$ seconds

Scanned with CamScanner