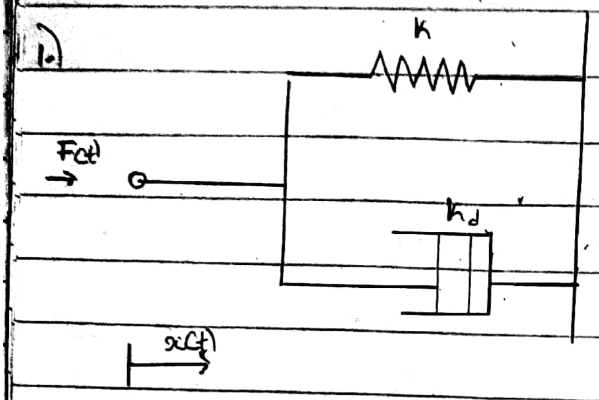
EKEOGU IZUCHUKWU
17/ENG04/019
ELECT/ELECT ENGR

EEE 324

1.)



$F_{spring} = Kx$

$F_{damper} = K_d \, dx/dt$

$F(t) = Kx + K_d \, dx/dt$

Using laplace transform

$F(s) = Kx(s) + K_d S \, x(s)$ ——— recall $d/dt = S$

$F(s) = [K + K_d S] \, x(s)$ - - - ①

$$G(s) = \frac{x(s)}{F(s)} = \frac{1/K}{Ts + 1} \quad \text{——— ②}$$

also

$$G(s) = \frac{x(s)}{F(s)} = \frac{1}{K + K_d S} \quad \text{——— ③}$$

comparing ② to ③

$Ts + 1 \Leftrightarrow K_d S + K$

$Ts + 1 \Leftrightarrow 1 + K_d S / K$

$$\therefore T\cancel{s} = \frac{K_d \cancel{S}}{K} = \frac{K_d}{K} = \frac{0.03}{4000} = 7.5 \times 10^{-6}$$

$$= \underline{7.5 \mu s}$$

a) Energy $= mc\Delta\theta$

$E_2 = mc(\theta - \theta_1)$

$E_1 = mc(\theta_2 - \theta_1)$

$G(s) = \frac{E_2}{E_1} = \frac{\cancel{m}c(\theta-\theta_1)}{\cancel{m}c(\theta_2-\theta_1)} = \frac{1}{Ts+1}$

$\theta - \theta_1 = \frac{\theta_2 - \theta_1}{Ts+1}$ Let $\theta_2 - \theta_1 = K(t)$

$\therefore \theta - \theta_1 = \frac{K(t)}{Ts+1} = \frac{K(1/T)}{s + 1/T}$ taking laplace transform. $K(t) = K/t$.

$\theta - \theta_1 = \frac{K(1/T)}{S(s+1/T)}$ inverse laplace

$\theta - \theta_1 = K(1-e^{-t/T})$ recall that $K = \theta_2 - \theta_1$

$\theta - \theta_1 = \theta_2 - \theta_1 (1-e^{-t/T})$

$\theta = (\theta_2 - \theta_1)(1-e^{-t/T}) + \theta_1$

$119 = (120-20)(1-e^{-360/T})$

$99 = 100 - 100e^{-360/T}$

$0.01 = e^{-360/T}$

$\ln 0.01 = -360/T$

$T = 78.17s$.

Thermal resistance $= T/C$

$C = mc = 0.5 \times 346 = 173 J/K$

$\therefore R = \frac{78.17}{173} = 0.452 KW$

$$\frac{\omega}{k_m x} = \frac{1}{Ts+1}$$

$$T = \frac{J}{K_3} \qquad k_m = \frac{k_1 k_2}{k_3}$$

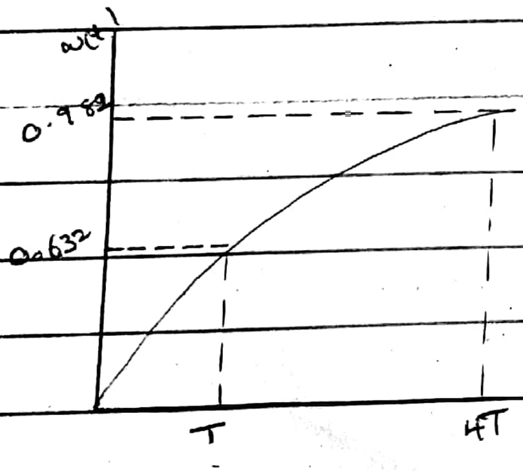
$$\omega = \frac{k_m x}{Ts+1}$$

Taking laplace transform of step input

$$\therefore \omega = \frac{k_m x}{s}\left[\frac{1}{Ts+1}\right]$$

$$= \frac{k_m x}{s}\left[\frac{1/T}{s+1/T}\right]$$

$$\omega(t) = k_m x\left\{1-e^{-t/T}\right\}$$



at $t=0$, $k_m x\left(1-e^{-t/T}\right) = 0$

at $t=T$, $k_m x\left(1-e^{-t/T}\right) = 0.632\, k_m x$

at $t=4T$, $k_m x\left(1-e^{-4T/T}\right) = 0.982\, k_m x$

For $t=T$,

$\Delta\% = (0.632-0)\times 100 = 63.2\%$

For $t=4T$

$\Delta\% = (0.982-0)\times 100 = 98.2\%$

$$G(s) = \frac{\theta_o}{\theta_i} = \frac{1}{3s+1}$$

$$\theta_o(s) = \frac{\theta_i(s)}{3s+1}$$

$$\theta_o(s) = \frac{c}{s^2(3s+1)} = \frac{c/3}{s^2(s+1/3)}$$

$$\frac{c/3}{s^2(s+1/3)} = \frac{As+B}{s^2} + \frac{C}{s+1/3}$$

$$c/3 = (As+B)(s+1/3) + Cs^2$$

$$\theta_o(t) = c[t - 3(1-e^{-t/3}) \quad \text{—— (1)}$$

at $t = 0$

$$\theta_o(t) = c[t - 3(1)]$$

$$= ct - 3c$$

$$\theta_e = \theta_i - \theta_o = \cancel{ct} - \cancel{ct} + 3c = 3c$$

$T = 3 \qquad c = 4\text{ mm/s}$

after 2 secs

$\theta_i = 4 \times 2 = 8\text{ mm}$

$\theta_e = c \times 3 = 4 \times 3 = 12\text{ mm}$

θ_o at (1)

$$\theta_o = 4[2 - 3(1 - e^{-2/3})] = 2.161$$

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ii) $G(s) = \dfrac{2}{0.2s + 0.5} = \dfrac{2/0.5}{\frac{0.2}{0.5}s + 1}$

compared to $\dfrac{K}{Ts + 1}$

DC gain = $\underline{\underline{2/0.5 = 4}}$ Time const = $T = 0.2/0.5 = \underline{\underline{0.4}}$

ii) $\dfrac{0.2}{0.05s + 0.1} = G(s) = \dfrac{0.2/0.1}{\frac{0.05}{0.1}s + 1}$

DC gain = $0.2/0.1 = 2$ Time const = $0.05/0.9 = \underline{\underline{0.5}}$

iii) $G(s) = \dfrac{2}{3s + 1}$

DC gain = 2 Time const = $\underline{\underline{3}}$

iv) $G(s) = \dfrac{16}{8s + 4} = \dfrac{16/4}{\frac{8}{4}s + 1}$

DC gain = $16/4 = \underline{\underline{4}}$ Time const = $8/4 = \underline{\underline{2}}$.

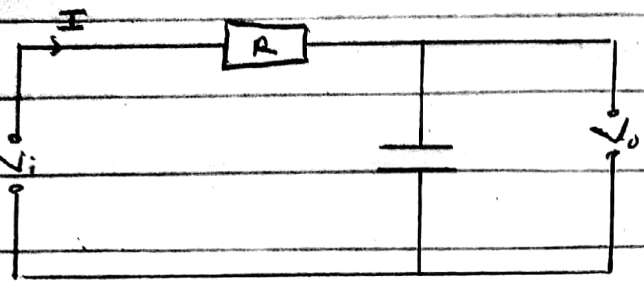
$\dfrac{\omega}{\theta}(s) = \dfrac{K_m}{T_m s + 2}$ if $K_m = 15 s^{-1}$ $T_m = 4$.

$= \dfrac{15}{4s + 2} = \dfrac{15/2}{\frac{4}{2}s + 1}$

[illegible] $15/2 = 7.5 s^{-1}$ Time const = $\underline{\underline{2 s}}$.

SYSTEM RESPONSE I

1)



$R = 47\,\Omega$ $C = 20\,\mu F$ $V_i = 5 \sin 2000t$

$G(s) = V_o(s)/V_i(s) = \frac{1}{Ts+1}$ where $T = RC$

$T = 47 \times 20 \times 10^{-6} = 9.4 \times 10^{-4}\,s$

$V_i = IR + I/C$

$V_o = I/Cs$

$\therefore G(s) = \frac{V_o}{V_i} = \frac{I/Cs}{IR + I/Cs} = \frac{1/Cs}{R + 1/Cs} = \frac{1}{RCs + 1}$

$\phi = -\tan^{-1}(\omega T) = -\tan^{-1}(2000 \times 9.40 \times 10^{-4}) = -42^\circ$

$$\frac{\theta_o}{\theta_i} = \frac{1}{\sqrt{1+T^2\omega^2}} = \frac{1}{\sqrt{1+(9.4\times10^{-4})^2(2000^2)}} = 0.47.$$

Therefore

$\theta_o = (5 \times 0.47) \sin(2000t - 62^\circ)$

$= 2.35 \sin 2000t - 62^\circ.$

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$x_o/x_i = 1/(T^2s^2 + 2\delta Ts + 1)$

$T = 0.4s \quad \delta = 0.2$

$\theta_i = 6\sin(\omega t)$ at 2.5 rad/s

$$C = \frac{(1-\omega^2T^2)}{\{(1-\omega^2T^2) + (2\delta\omega T)^2\}}$$

$$C = \frac{1-2.5^2\times 0.4^2}{\{(1-(2.5^2\times 0.4^2) + (2\times 0.2\times 2.5\times 0.4)^2\}}$$

$C = 0$

$$D = \frac{2\delta T\omega}{\{(1-T^2\omega^2) + (2\delta\omega T^2)^2}$$

$$= \frac{2\times 0.2\times 0.4\times 2.5}{\{(1-0.4^2\times 2.5^2)^2 + (2\times 0.2\times 2.5\times 0.4)^2\}}$$

$$= \frac{0.4}{0+0.4^2}$$

$D = 2.5$

$\phi = -\tan^{-1}(D/C) = -\tan^{-1}(\infty) \therefore \theta = 90$

$|\theta_o/\theta_i| = \sqrt{D^2+C^2}$

$= \sqrt{2.5^2+0^2}$

$= 2.5$

$\theta = 6\times 2.5$

$= 15°$ //