4) $\frac{\theta_o}{\theta_i}(s) = \frac{1}{3s+1}$

$\theta_o(s) = \frac{\theta_i(s)}{3s+1}$

$\theta_o(s) = \frac{C}{s^2(3s+1)}$

$\theta_o(t) = \frac{C/3}{s^2(s+1/3)}$

$\theta(t) = C(t - 3(1 - e^{-t/3}))$

Where $t \geq 6s$

$\theta_o(t) = C(t - 3(1))]$

$\theta_o(t) = Ct - 3C$

$\theta_i - \theta_o = \theta_o = Ct - C(t - 3C) = 3C$

$T = 3 \quad C = 4mm/s$

after 2 seconds

$\theta = 4 \times 2 = 8mm$

$\theta_o = 4mm \times 3 = 12mm$ at steady state

$\theta_o = 4(2 - 3(1 - e^{2/3})$

$\cancel{= 2 \ldots} = 2.161 mm$

1) From the diagram

$Spring = k(x-0)$

$f_{damper} = kd \frac{d(x-0)}{dt}$

$f(t) \Rightarrow f(t)$

$f(t) - k(x-0) - kd\frac{d(x-0)}{dt}$

$0 = f(t) - kx - kd\frac{dx}{dt}$

Taking the laplace transform

$\therefore f(s) = kx(s) - kds\,x(s) = 0$

$f(s) = (k + kds)\,x(s)$

$G(s) = \frac{x(s)}{f(s)} = \frac{1}{k + kds}$

$\Rightarrow \frac{1/k}{1 + [kd/k]s}$

ii)

6 $\frac{\omega}{\theta}(s) = \frac{K_m}{T_m s + 2}$

$K_m = 15 s^{-1}$

$T_m = 4$

$= \frac{15}{4s+2} = \frac{15/2}{4s/2 + 2/2}$

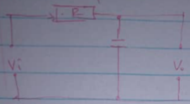
$= \frac{15/2}{4s/2 + 1} = \frac{7.5}{2s+1}$

Dc gain $= 7.5 \, ms^{-1}$

Time constant $= 2$ seconds

System Response II

1)



$T = R_C$

$R = 47\Omega \quad C = 20\mu F$

$V_i = 5\sin(2000t)$

$T = 47 \times 20 \times 10^{-6} = 9.4 \times 10^{-9}$

$\left(\frac{V_o}{V_i}\right)(s) = \frac{1}{Ts+1}$

$G(s) = \frac{1}{Ts+1}$

$G(\omega) = \frac{1}{9.4\times10^{-9}j\omega+1} \times \frac{9.4\times10^{-9}j\omega-1}{9.4\times10^{-9}j\omega-1}$

$G(\omega) = \frac{9.4\times10^{-9}j\omega-1}{(9.4\times10^{-9})^2\omega^2-1}$

$G(\omega) = \frac{-1}{(9.4\times10^{-9})\omega^2-1}$

where $\omega = 2000$ rad/s

$\phi = \tan^{-1}\left(\frac{9.4\times10^{-1}(2000)^2-1}{1}\right)$

$\phi = -61.99$

2) E_2 = new energy

E_1 = initial energy

$$E_2 = mc\Delta\theta \Rightarrow E_2 = mc(\theta - \theta_1)$$

$$E_1 = mc\Delta\theta \Rightarrow E_2 = mc(\theta_2 - \theta_1)$$

where θ is the new temp of the metal

$$G(s) = \frac{E_2}{E_1} = \frac{mc(\theta - \theta_1)}{mc(\theta_2 - \theta_1)} = \frac{1}{T_s + 1}$$

$$\Rightarrow \frac{\theta - \theta_1(s)}{\theta_2 - \theta_1} = \frac{1}{T_s + 1}$$

$$\theta - \theta_1 = \frac{\theta_2 - \theta_1}{T_s + 1}$$

let $\theta_2 - \theta_1(s) = k(t)$

$$(\theta - \theta_1)(s) = \frac{k(t)}{T_s + 1}$$

$$(\theta - \theta_1)(s) = \frac{k(t)(1/T)}{s + 1/T}$$

taking the laplace transform of $k(t)$

$$= \frac{k}{s}$$

$$(\theta - \theta_1)(s) = \frac{k(1/T)}{s(s + 1/T)}$$

$$G(j\omega) = \frac{1}{\sqrt{(9.4\times10^{-4})^2\omega^2 + 1^2}}$$

$$= \frac{1}{\sqrt{((9.4)\times10^{-4})^2(2000)^2 + 1^2}}$$

$$= 0.4696$$

$$V^{o} = 5 \times 0.4696 = 2.35$$

2) $$\frac{x_0}{x_i} = \frac{1}{T^2s^2 + 2\delta T_s + 1}$$

$$G(s) = \frac{1}{(1 - T^2s^2) + 2\delta T_s}$$

$$G(j\omega) = \frac{1}{(1 - T^2\omega^2) + 2\delta T j\omega}$$

$\delta = 0.2$ $T = 0.45$ $\omega = 2.5$ rad/s

$$G(j\omega) = \frac{1 - T^2\omega^2 - 2\delta T j\omega}{(1 - T^2\omega^2) + 4\delta^2 T^2\omega^2}$$

$$= \frac{1 - (0.4)^2(2.5)^2 - 2(0.2)(0.4)(2\pi)}{(1 - (0.4)^2(2.5)^2) - 4(0.2)^2(0.4)(2.5)^2}$$

$$G(j\omega) = 0 - 2.5$$

$$\phi = \tan^{-1}\left(\frac{2.5}{0}\right) = t$$

$$\tan^{-1}(\infty) = 90^{\circ}$$

$$|G(j\omega)| = \sqrt{0^2 + (2.5^2)}$$

$$= 2.5$$

$$G(j\omega) = \frac{1}{\sqrt{(7.4\times10^{-4})^2\omega^2 + 1^2}}$$

$$= \frac{1}{\sqrt{((7.4)\times10^{-4})^2(3000)^2 + 1^2}}$$

$$= 0.4696$$

$$V^o = 5 \times 0.4696 = 2.35$$

2) $$\frac{x_o}{x_i} = \frac{1}{T^2S^2 + 2\delta T_s + 1}$$

$$G(s) = \frac{1}{(1 - T^2S^2) + 2\delta T_s}$$

$$G(j\omega) = \frac{1}{(1 - T^2\omega^2) + 2\delta T j\omega}$$

$\delta = 0.2$ $T = 0.4s$ $\omega = 2.5$ rad/s

$$G(j\omega) = \frac{1 - T^2\omega^2 - 2\delta T j\omega}{(1 - T^2\omega^2) + 4\delta^2 T^2\omega^2}$$

$$= \frac{1 - (0.4)^2(2.5)^2 - 2(0.2)(0.4)(2\pi)}{(1 - (0.4)^2(2.5)^2) - 4(0.2)^2(0.4)(2.5)^2}$$

$$G(j\omega) = 0 - 2.5$$

$$\phi = \tan^{-1}\left(\frac{2.5}{0}\right) = t$$

$$\tan^{-1}(\infty = 90^\circ$$

$$|G(j\omega)| = \sqrt{0^2 + (2.5^2)}$$

$$= 2.5$$

5) $\frac{2}{0.2s+0.5} = \frac{2/0.5}{0.2s/0.5+1}$

$\Rightarrow \frac{4}{0.4s+1}$

4 = DC gain

0.4 = Time constant

ii) $\frac{0.2}{0.05s+0.1} = \frac{0.2/0.1}{0.05s/0.1+1}$

$= \frac{2}{0.5s+1}$

2 = DC gain

0.5 = time constant

iii) $\frac{2}{3s+1}$

2 = DC gain

3 = Time constant

iv) $\frac{16}{8s+4} = \frac{16/4}{8/4s+1}$

$= \frac{4}{2s+1}$

4 = DC gain

2 = Time constant

Inverse laplace transform

$$(\theta - \theta_1) = k\left[1 - e^{-t/T}\right]$$

from (1) $k = \theta_2 - \theta_1$

$$(\theta - \theta_1) = (\theta_2 - \theta_1)\left[1 - e^{-t/T}\right]$$

3) $\frac{w}{kmx} = \frac{1}{Ts+1}$

$T = 1/ks$ $\quad km = \frac{k_1 k_2}{k_s}$

$w = \frac{k_m x}{Ts+1}$

Laplace Transform of the step input

$w = \frac{k_m x}{s}\left(\frac{1}{Ts+1}\right)$

$\frac{k_m x}{s}\left(\frac{1/T}{s+1/T}\right)$

$w(s) \Rightarrow k_m x\,(1 - e^{-t/T})$

at $t = 0$ $k_m x\,(1 - e^{0}) =$ Initial

at $t = T$ $k_m x\,(1 - e^{-1 T/T}) = 0.63\, k_m x$

at $t = 4T = k_m x\,(1 - e^{-4T/T})$

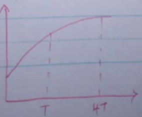
$= 0.981\, k_m x$

for $t = T$

$\Delta\% = (0.632 - 0) \times 100\% = 63.2\%$

$t = 4T$

$\Delta\% = (0.981 - 0) \times 100\% = 98.1\%$



Amplitude $\Rightarrow 6 \times 2.5$
$= 15$