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Mechatronics Engineering
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MAT 102

1. $M = Pi - 6j - 3k$, $N = 4i + 3j - k$, $O = i - 3j + 2k$

a. M and N are perpendicular

$\bar{M} \cdot \bar{N} = 0$

$\bar{M} \cdot \bar{N} = (Pi - 6j - 3k) \cdot (4i + 3j - k)$

$= 4p - 18 + 3$

for perpendicular vector

$4p - 18 + 3 = 0$

$4p - 15 = 0$

$\frac{4p}{4} = \frac{+15}{4}$

$P = 3\frac{3}{4}$

b. M, N and O are coplanar

$\bar{A} \cdot (\bar{B} \times \bar{C}) = 0$

$$\bar{A} \cdot (\bar{B} \times \bar{C}) = \begin{vmatrix} + & - & + \\ P & -6 & -3 \\ 4 & 3 & -1 \\ 1 & -3 & 2 \end{vmatrix}$$

$$P \begin{vmatrix} 3 & -1 \\ -3 & 2 \end{vmatrix} - \left(-6 \begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix}\right) + \left(-3 \begin{vmatrix} 4 & 3 \\ 1 & -3 \end{vmatrix}\right)$$

$P(6 - (3)) + 6(8 - (-1)) - 3(-12 - 3)$

$P(3) + 6(9) - 3(-15)$

$= 3p + 54 + 45$

$\bar{A} \cdot (\bar{B} \times \bar{C}) = 0$

$3p + 54 + 45 = 0$

$3p + 99 = 0$

$3p = -99$

$p = -33$

2 Find the direction cosines and the unit vector along the sum of $3i + 2j + 5k$, $2i - j + 6k$ and $5i + 2j - 3k$.

- $3i + 2j + 5k$

$a_x = 3$ $a_y = 2$ $a_z = 5$

$= \sqrt{3^2 + 2^2 + 5^2}$

$= \sqrt{38}$

Direction cosines are.

$\cos\alpha = \frac{3}{\sqrt{38}} = 0.487$

$\cos\beta = \frac{2}{\sqrt{38}} = 0.324$

$\cos\gamma = \frac{6}{\sqrt{38}} = 0.811$

- $2i - j + 6k$

$a_x = 2$ $a_y = -1$ $a_z = 6$

$|| = \sqrt{2^2 + (-1)^2 + 6^2} = \sqrt{41}$

direction cosines are:

$\cos\alpha = \frac{2}{\sqrt{41}} = 0.312$

$\cos\beta = \frac{-1}{\sqrt{41}} = -0.156$

$\cos\gamma = \frac{6}{\sqrt{41}} = 0.937$

- $5i + 2j - 3k$

$a_x = 5$ $a_y = 2$ $a_z = -3$

Modulus $= \sqrt{5^2 + 2^2 + (-3)^2} = \sqrt{38}$

Direction cosines are!

$\cos\alpha = \frac{5}{\sqrt{38}} = 0.811$

$\cos\beta = 0.324 = \frac{2}{\sqrt{38}}$

$\cos\gamma = \frac{-3}{\sqrt{38}} = -0.487$

Unit vector

$(3i + 2j + 5k) + (2i - j + 6k) + (5i + 2j - 3k)$

$= 10i + 3j + 8k$

$e_{A+B+C} = \frac{\overline{A+B+C}}{|A+B+C|}$

$|A+B+C| = \sqrt{10^2 + 3^2 + 8^2} = \sqrt{173}$

$e_{A+B+C} = \frac{10i + 3j + 8k}{\sqrt{173}}$

$e_{A+B+C} = \frac{10i}{\sqrt{173}} + \frac{3j}{\sqrt{173}} + \frac{8k}{\sqrt{173}}$

3. $f = 3ui + u^2 j + (u+2)k$ and $v = 2ui - 3uj + (u-2)k$

$(f \times v) = \begin{vmatrix} i & j & k \\ 3u & u^2 & (u+2) \\ 2u & -3u & (u-2) \end{vmatrix}$

$i \begin{vmatrix} u^2 & (u+2) \\ -3u & (u-2) \end{vmatrix} - j \begin{vmatrix} 3u & (u+2) \\ 2u & (u-2) \end{vmatrix} + k \begin{vmatrix} 3u & u^2 \\ 2u & -3u \end{vmatrix}$

$i[u^3 - 2u^2 - (-3u^2 - 6u)] - j[3u^2 - 6u - (2u^2 + 4u)] + k(-9u^2 - 2u^3)$

$(u^3 - u^2 + 6u)i \cancel{+} j(u^2 - 10u) + k(-9u^2 - 2u^3)$

$$\int_0^1 (u^3-u^2+6u)i-(u^2-10u)j+(-9u^2-2u^3)k\,du$$

$$i\left(\frac{u^4}{4}-\frac{u^3}{3}+\frac{6u^2}{2}\right)\Big|_0^1 - j\left(\frac{u^3}{3}-\frac{10u^2}{2}\right)\Big|_0^1 + k\left(\frac{-9u^3}{3}\ldots\right.$$

$$\left(\frac{u^4}{4}-\frac{u^3}{3}+3u^2\right)_i\Big|_0^1 - \left(\frac{u^3}{3}-5u^2\right)_j\Big|_0^1 + \left(-3u^3+\frac{u^4}{2}\right)_k$$

$$i\left[\left(\frac{1^4}{4}-\frac{1^3}{3}+3\times 1^2\right)-\left(\frac{0^4}{4}-\frac{0^3}{3}+(3\times 0)^2\right)\right]+j\left[\left(\frac{1^3}{3}-5\times 1^2\right)-\right.$$

$$\left.\left(\frac{0^3}{3}-5\times 0^2\right)\right]+k\left[\left((-3\times 1^3)+\frac{1^4}{2}\right)-\left(-3\times 0^3+\frac{0^4}{2}\right)\right]$$

$$= i\left[\frac{1}{4}-\frac{1}{3}+9\right]-0\Big)+j\left[\left(\frac{1}{3}-25\right)-0\right]+k\left(\left(-27+\frac{1}{2}\right)\right.$$

$$\left.-0\right]$$

$$= i\left[\frac{107}{12}\right]+j\left[-\frac{74}{3}\right]+k\left(-\frac{53}{2}\right)$$

$$= \frac{107}{12}i - j\left(\frac{74}{3}\right) - \frac{53}{2}k$$

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