Ejike David Chinedu
18/EN904/078
Electrical/Electronics Engineering
Linear System Assignment

(a) Find the DC gain and the time constant for the following transfer functions

(i) $G(s) = \frac{2}{0.2s + 0.5}$

Take s to be 0

$= \frac{2}{0.2 \times 0 + 0.5}$

Take s to be 0

$= \frac{2}{0.2 \times 0 + 0.5} = \frac{2}{0.5}$

DC gain = 4

Time constant: $\frac{2/0.5}{(0.5/0.2)s + 1}$

$= \frac{4}{0.4s + 1}$

Time constant = 0.4

(ii) $G(s) = \frac{0.2}{0.05s + 0.1}$

If s = 0

DC gain $= \frac{0.2}{0.1} = 2$

Time constant $= \frac{0.05}{0.1} = 0.5$

Scanned with CamScanner

(iii) $G(s) = \dfrac{2}{3s+1}$

D.C gain = 2

Time constant = 3

(iv) $G(s) = \dfrac{16}{8s+4}$

Time constant = $8/4 = 2$ $(J/B)$

Gain = $16/4 = 4 (T/B)$

(6) $\dfrac{\omega}{\theta}(s) = \dfrac{km}{Tms+2}$

$Km = 15s^{-1}$ and $Jm = 4s$

$\omega/\theta (s) = \dfrac{15}{4s+2}$

Gain = $15/2 = km/B = 7.5s^{-1}$

Time constant = $4/2 = T/B$ = 2 seconds

(4) $\dfrac{\theta_o(s)}{\theta_i} = \dfrac{1}{Ts+1}$

$\theta_i(t) = ct$

$\theta_i(s) = \dfrac{c/s^2}{(Ts+1)}$

$= c/s^2/(Ts+1)$

$= \dfrac{c}{s^2(Ts+1)}$

Scanned with CamScanner

$\theta_o(t) = C(t - T(1 - e^{-t/T}))$

$(\theta_e(t)) = CT$

$G(s) = 1/3(s+1)$ $\quad C = 4 \quad t = 2$

After 2 seconds

$\theta_i = 4 \times 2 = 8mm$

$\theta_o = 4(2 - 3(1 - e^{-2/3}))$

$= 4(2 - 3(0.486))$

$= 4(2 - 1.459)$

$= 4(0.540)$

$= 2.161mm$

$\theta_e = 8 - 2.161$

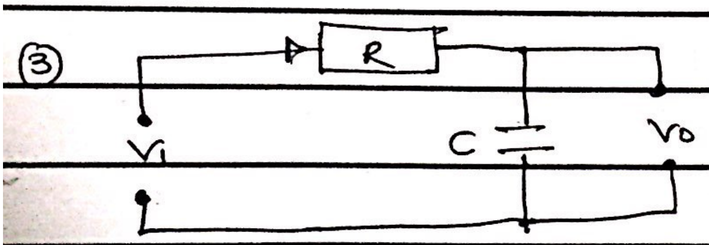
$= 5.838mm$

steady state error is CT

$= 4 \times 3$

$= 12mm$

(3)




$G(s) = \frac{Vo(s)}{Vi} = \frac{1}{Ts+1}$ where $T = RC$

$R = 47\Omega$ and $C = 20\mu F$ $\quad Vi = 5\sin(200t)$

$V_i = IR + I/Cs$

$V_i = I(R + 1/Cs)$

$V_o = I/Cs$

$G(s) = Vo/V_i(s) = \frac{1/Cs}{1(R + 1/Cs)} = \frac{1}{RCs+1}$

$T = RC$

$G(s) = \frac{1}{Ts + 1}$

$T = 47 \times 20 \times 10^{-6} = 940 \times 10^{-6} s$

$\phi = -\tan^{-1}(\omega t) = -\tan^{-1}(2000 \times 940 \times 10^{-6}) = -62^\circ$

$\frac{\theta_o}{\theta_i} = \frac{1}{\sqrt{1 + T^2\omega^2}} = \frac{1}{\sqrt{1 + (940 \times 10^{-6})^2 \times 2000^2}}$

Therefore

$\theta_o = (5 \times 0.47)\sin(2000t - 62^\circ)$

$= 2.35\sin(2000t - 62^\circ)$

(2) $X_o / X_i = 1/(T^2s^2 + 2\delta Ts + 1)$

$T = 0.4$ seconds

$\delta = 0.2$

$\theta_1 = 6\sin(\omega t)$ at $2.5$ rad(s)

Amplitude $= ?$

Phaseshift $= ?$

Constants C and D

$C = \frac{(1 - \omega^2T^2)}{\{(1 - \omega^2T^2) + (2\delta\omega T)^2\}}$

$= \frac{(1 - 2.5^2 \cdot 0.4^2)}{\{(1 - 2.5^2 0.4^2) + (2 \times 0.2 \times 2.5 \times 0.4)^2\}}$

$C = \frac{0}{\{0 + (0.4)^2\}}$

$C = 0$

$D = \frac{2\delta T\omega}{\{(1 - T^2\omega^2)^2 + (2\delta\omega T)^2\}}$

$$= \frac{2 \times 0.2 \times 0.4 \times 2.5}{\{(1-0.4^2 \times 2.5^2) + (2 \times 0.2 \times 2.59 \times 0.4)^2\}}$$

$$= \frac{0.4}{\{0 + 0.4\}^2}$$

$$= \frac{0.4}{0.16} = 2.5$$

$\theta = \tan^{-1}(\infty) = \tan^{-1}(\infty)$ therefore $\theta = 90°$

$$|\theta_o/\theta_i| = \sqrt{D^2 + C^2}$$

$$= \sqrt{2.5^2 + 0^2}$$

$$= 2.5$$

$$\theta = 6 \times 2.5 = 15$$

(1) $kd = 0.03$ $\quad T = 100N$

$K = 4KN/m$ $\quad x = 16mm$

$$G(s) = \frac{x(s)}{F(s)} = \frac{1/k}{Ts+1}$$

$$F = kx + kd\, dx/dt \rightarrow (1)$$

$$F = \frac{B\,dx}{dt} + Kx$$

Laplace transform of eqn(1)

$$F(s) = Kx + kdsx$$

$$F(s) = x(K + kds)$$

$$\frac{F(s)}{x} = k + kds$$

Taking inverse

$$\frac{x}{F(s)} = \frac{1}{k + k_d s}$$

$$= \frac{1}{k_d s + k}$$

$$\frac{X}{F(s)} = \frac{1/k}{(k_d/k)s + 1} = \frac{1/k}{Ts + 1}$$

$$T = k_d / k$$

$$= \frac{0.03}{4 \times 10^3} = 7.5 \times 10^{-6} \text{ seconds}$$

$X_o$ after $T$ seconds

$$X_o = F/k\,(1 - e^{-1})$$

$$= 100/4 \times 10^3\,(1 - e^{-1})$$

$$= 100/4 \times 10^3 \times 0.632$$

$$= 0.025 \times 0.632$$

$$= 0.0158\,m$$

$$= 0.016\,m \approx 16 \times 10^{-3} = 16\,mm$$

(2) $M = 0.5\,kg$

specific capacity $= 346 \cdot \theta_i = 20^\circ$

$\theta_2 = 120^\circ C$

$\theta = 119^\circ$

$\theta_1 / \theta_2(s) = 1/(Ts + 1)$

show that $\theta = \theta_1 + (\theta_2 - \theta_1)(1 - e^{-t/T})$

$$\frac{\theta}{\theta_2} = \frac{1}{Ts + 1}$$

$$\theta = Ts\theta_2 + \theta_2$$

$\theta = \theta_2 (\tau s + 1)$

$\theta_{(s)} = \dfrac{\theta_2}{\tau s + 1}$

$\theta_t = \dfrac{1}{s(\tau s + 1)}$ → Inverse Laplace transform

$\theta_t = \dfrac{1/\tau}{s(s + 1/\tau)}$

$\theta(t) = 1 - e^{-t/\tau}$

$\theta(t) = 1 - e^{-t/\tau}$

$\theta(t) = \theta_1 + (\theta_2 - \theta_1)(1 - e^{-t/\tau})$

$119 = 20 + (120 - 20)(1 - e^{-6/\tau})$

$99 = 100(1 - e^{-6/\tau})$

$0.99 = (1 - e^{-6/\tau})$

$-0.01 = e^{-6/\tau}$

$\ln 0.01 = -6/\tau$

$-4.605 = -6/\tau$

$\tau = 6/4.605$

$= 1.302$ mins or $78.17$ sec

Thermal capacitance $C = mC = 0.5 \times 346 = 173$ J/k.

$\tau = RC$

$R = \tau/C$

$R = 78.17/173 = 0.451$ K/W

Scanned with CamScanner

(3) $\frac{\omega}{K_m x} = \frac{1}{T\sigma + 1}$

$T = \frac{J}{K_3}$ $\quad K_m = \frac{K_1 K_2}{K_3}$

$K_1, K_2, x, \omega, J, K_3, K_4$

Assuming no load torque

Torque $= K \cdot P$

$= Ja + K_3\omega$

$K_1 K_2 x(s) = J(s)\omega' + K_3\omega$

$(K_1 K_2 / K_3) x(s) = \omega\{(J/K_3)s + 1\}$

$\frac{\omega}{K_m x} = \frac{1}{s T\sigma + 1}$

Rearrang

$\frac{1/4}{s(s + 1/4)}$

$\omega / K_m (t) = (1 - e^{-t/T})$

sub $t (= T\ \omega(t)) =$

$K_m (1 - e^{-1}) = 0.632 K_m$

$= 0.632 \rightarrow 63.2\%$ change

Subt $= 4T\ \omega(t) = K_m (1 - e^{-4}) = 0.982 K_m$

$= 98.2\%$ change