

MAT 104 Assignment

Answers

$$1.) y = \frac{2 \cos 3x}{x^3}$$

$$\text{Let } u = 2 \cos 3x, \quad v = x^3$$

$$\frac{du}{dx} = -6 \sin 3x, \quad \frac{dv}{dx} = 3x^2$$

$$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

$$\Rightarrow \frac{x^3(-6 \sin 3x) - 2 \cos 3x(3x^2)}{x^6}$$

$$\therefore \frac{dy}{dx} = \frac{-6x^2(x \sin 3x - \cos 3x)}{x^6}$$

$$2.) y = x e^{2x}$$

$$\text{Show that } \frac{d^2 y}{dx^2} - 4 \frac{dy}{dx} + 4y = 0$$

$$\text{Let } u = x, \quad v = e^{2x}$$

$$\frac{du}{dx} = 1, \quad \frac{dv}{dx} = 2e^{2x}$$

$$\therefore \frac{dy}{dx} \Rightarrow e^{2x} + 2x e^{2x}$$

$$4) \int e^x \sin 2x$$

$$v = e^{2x} \quad \text{and} \quad du = \sin 2x$$

$$dv = e^{2x} dx, \quad u = -\frac{\cos 2x}{2}$$

$$\int v du = uv - \int u dv$$

$$\int e^x \sin 2x = -\frac{e^x \cos 2x}{2} + \int \frac{e^x \cos 2x}{2}$$

$$\int \frac{e^x \cos 2x}{2} = \frac{1}{2} \int e^x \cos 2x$$

$$\int e^x \cos 2x = \left(\frac{e^x \sin 2x}{2} - \int \frac{e^x \sin 2x}{2} \right) \cdot \frac{1}{2}$$

$$\therefore \int e^x \sin 2x = \frac{e^x \sin 2x}{4} - \frac{e^x \cos 2x}{2} - \left(\frac{e^x \sin 2x}{4} \right)$$

Let $\int e^x \sin 2x$ be y

$$y = \frac{e^x \sin 2x}{4} - \frac{e^x \cos 2x}{2} - \frac{y}{4} + C$$

$$5y = \frac{e^x \sin 2x}{4} - \frac{e^x \cos 2x}{2} + C$$

$$5y = \frac{e^x \sin 2x}{4} - 2 \frac{e^x \cos 2x}{2} + C$$

$$y = \frac{e^x \sin 2x}{5} - 2 \frac{e^x \cos 2x}{5} + \frac{C}{5}$$

Putting the value of y back

$$\therefore \int e^x \sin 2x = \frac{e^x \sin 2x}{5} - 2 \frac{e^x \cos 2x}{5} + \frac{C}{5}$$

$$\frac{dy}{dx} = e^{2x} + 2xe^{2x}$$

Let $u =$

$$\frac{dy}{dx} = e^{2x} + 2xe^{2x}$$

$$\frac{d^2y}{dx^2} = 2e^{2x} + \frac{d}{dx}(2xe^{2x})$$

$$= 2e^{2x} + 2e^{2x} + 4xe^{2x}$$

$$\therefore \frac{d^2y}{dx^2} = 4e^{2x} + 4xe^{2x}$$

$$4 \frac{dy}{dx} = 4(e^{2x} + 2xe^{2x}) \Rightarrow 4e^{2x} + 8xe^{2x}$$

$$4y = 4(xe^{2x}) \Rightarrow 4xe^{2x}$$

$$\text{Then } \frac{d^2y}{dx^2} - 4 \frac{dy}{dx} + 4y = 0$$

$$(4e^{2x} + 4xe^{2x}) - (4e^{2x} + 8xe^{2x}) + 4xe^{2x} = 0$$

$$\therefore \frac{d^2y}{dx^2} - 4 \frac{dy}{dx} + 4y = 0 \quad \text{Is correct}$$