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Q1

$$\int \frac{3x-1}{(x-1)(x-2)(x-3)} dx = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$$

Solve the partial fraction

$$3x-1 = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)$$

$$3x-1 = A(x^2-5x+6) + B(x^2-4x+3) + C(x^2-3x+2)$$

$$3x-1 = Ax^2 - 5Ax + 6A + Bx^2 - 4Bx + 3B + Cx^2 - 3Cx + 2C$$

$$3x-1 = Ax^2 + Bx^2 + Cx^2 - 5Ax - 4Bx - 3Cx + 6A + 3B + 2C$$

$$3x-1 = (A+B+C)x^2 + (-5A-4B-3C)x + (6A+3B+2C)$$

$$3x = (-5A-4B-3C)x \quad (\text{for } x)$$

$$3 = -5A - 4B - 3C$$

$$0 = 6A + B + C \quad (\text{for } x^2)$$

$$0 = A + B + C$$

$$-1 = 6A + 3B + 2C \quad (\text{for } x^0)$$

$$3 = -5A - 4B - 3C \quad \textcircled{1}$$

$$0 = A + B + C \quad \textcircled{2}$$

$$-1 = 6A + 3B + 2C \quad \textcircled{3}$$

$$A, B, C = C1, -5, 4$$

$$= \int \frac{1}{x-1} - \frac{5}{x-2} + \frac{4}{x-3} dx$$

$$= \int \frac{1}{x-1} dx - \int \frac{5}{x-2} dx + \int \frac{4}{x-3} dx$$

$$= \ln|x-1| - 5 \ln|x-2| + 4 \ln|x-3|$$

$$= \ln|x-1| - 5 \ln|x-2| + 4 \ln|x-3| + C, C \in \mathbb{R}$$



②

$$\int \frac{x^2 + x + 1}{(x+2)(x^2+1)} dx$$

Solve into partial fraction

$$= \frac{A}{x+2} + \frac{Bx+C}{x^2+1}$$

$$= x^2 + x + 1 = (x^2+1)A + (x+2)(Bx+C)$$

$$= x^2 + x + 1 = Ax^2 + A + Bx^2 + Cx + 2Bx + 2C$$

$$x^2 + x + 1 = Ax^2 + Bx^2 + Cx + 2Bx + A + 2C$$

$$x^2 + x + 1 = (A+B)x^2 + (C+2B)x + A+2C$$

$$1 = A + 2C \quad \text{--- ①}$$

$$1 = C + 2B \quad \text{--- ②}$$

$$1 = A + B \quad \text{--- ③}$$

$$A, B, C = \left( \frac{3}{5}, \frac{2}{5}, \frac{1}{5} \right)$$

$$= \int \frac{A}{x+2} + \frac{Bx+C}{x^2+1}$$

$$= \int \frac{3}{5(x+2)} + \frac{2x+1}{5(x^2+1)}$$

$$= \int \frac{3}{5(x+2)} + \frac{x^2+1}{5(x^2+1)} dx$$

~~$$= \frac{x^3 + 3x^2 + 6x + 1}{6} + \frac{1}{2} \ln|x+2| + \frac{1}{2} \ln|x^2+1| + \frac{1}{2} \ln|x^2+1| + \frac{1}{2} \ln|x^2+1| + \frac{1}{2} \ln|x^2+1| + \frac{1}{2} \ln|x^2+1| + \frac{1}{2} \ln|x^2+1| + \frac{1}{2} \ln|x^2+1| + \frac{1}{2} \ln|x^2+1| + \frac{1}{2} \ln|x^2+1|$$~~

~~$$= \frac{2x^3 + 6x^2 + 8x + 1}{6} + \frac{1}{2} \ln|x^2+1|$$~~

~~$$= \frac{2x^3 + 6x^2 + 8x + 1}{6} + \frac{1}{2} \ln|x^2+1| + C, C \in \mathbb{R}$$~~

$$= \int \frac{3}{5(x+2)} dx + \int \frac{2x+1}{5(x^2+1)} dx$$

$$= \frac{3}{5} \times \ln|x+2| + \frac{1}{5} \times (x^2+1) + \tan^{-1}(x)$$

$$= \frac{3}{5} \times \ln|x+2| + \frac{1}{5} (x^2+1) + \frac{\tan^{-1}(x)}{5} + C, C \in \mathbb{R}$$

③



⑧

$$\int \frac{x^2+1}{(x-3)(x-2)} dx$$

$$= \int \frac{x^2+1}{(x-3)(x-2)} dx$$

$$= \int \frac{10}{x-3} - \frac{9}{x-2} - \frac{5}{(x-2)^2} dx$$

$$= \int \frac{10}{x-3} dx - \int \frac{9}{x-2} dx - \int \frac{5}{(x-2)^2} dx$$

$$= 10 \ln|x-3| - 9 \ln|x-2| + \frac{5}{x-2}$$

$$= 10 \ln|x-3| - 9 \ln|x-2| + \frac{5}{x-2} + C, C \in \mathbb{R}$$



(4)

$$\int \frac{(x^3 + x^2 + x + 1)}{(x-1)} dx$$

$$= \int \frac{x^3}{x-1} + \frac{x^2}{x-1} + \frac{x}{x-1} + \frac{1}{x-1} dx$$

$$= \int \frac{x^3}{x-1} dx + \int \frac{x^2}{x-1} dx + \int \frac{x}{x-1} dx + \int \frac{1}{x-1} dx$$

$$= \frac{2x^3 + 3x^2 + 6x + 11}{6} + \ln|x-1| + \int \frac{x^2}{x-1} + \int \frac{x}{x-1} + \int \frac{1}{x-1}$$

$$= \frac{2x^3 + 3x^2 + 6x + 11}{6} + \ln|x-1| + \frac{x^2}{2} + x + \ln|x-1| + \int \frac{x}{x-1} + \int \frac{1}{x-1}$$

$$= \frac{2x^3 + 3x^2 + 6x + 11}{6} + \ln|x-1| + \frac{x^2}{2} + x + \ln|x-1| + x-1 + \ln|x-1| + \ln|x-1|$$

$$= \frac{2x^3 + 6x^2 + 18x - 17}{6} + 4\ln|x-1|$$

$$= \frac{2x^3 + 6x^2 + 18x - 17}{6} + 4\ln|x-1| + C, C \in \mathbb{R}.$$