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Mechatronics Engineering

Math 102

1. If $M = Pi - 6j - 3k$ $N = 4i + 3j - k$ $O = i + 3j + 2k$ find the value of which (a) M and N are perpendicular to each other.
(b) M, N, and O are coplanar

Soln

(a) Perpendicular vectors = 0

$\therefore M \cdot N = 0$

$0 = (Pi - 6j - 3k) \cdot (4i + 3j - k)$

$0 = 4p - 18 + 3$

$0 = 4p - 15$

$15 = 4p$

$p = 15/4 \therefore p = 3\,3/4$

(b) $M \cdot (N \times O) = 0$

$M \cdot (N \times O) = \begin{vmatrix} p & -6 & -3 \\ 4 & 3 & -1 \\ 1 & -3 & 2 \end{vmatrix}$

$= P \begin{vmatrix} 3 & -1 \\ -3 & 2 \end{vmatrix} + 6 \begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix} - 3 \begin{vmatrix} 4 & 3 \\ 1 & -3 \end{vmatrix}$

$= P(6-3) + 6(8+1) - 3(12-3$

$= 3P + 99$

$0 = 3P + 99$

$-3P = 99$

$P = -99/3$

$P = -33$

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(2) find the direction cosine and the unit vector along the sum of $3i+2j+5k$, $2i+j+6k$, $5i+2j-3k$

solution

i.) Direction cosines = $\cos\theta = \dfrac{\bar{A}\ \bar{B}\ \bar{C}}{|A||B||C|}$

$(\bar{A}\cdot\bar{B}\cdot\bar{C}) = (3i+2j+5k)\cdot(2i-j+6k)\cdot(5i+2j-3k)$

$= 30 - 4 - 90$

$(\bar{A}\cdot\bar{B}\cdot C) = -64$

$\cancel{A}\ |A| = \sqrt{(3)^2+(4)^2+(5)^2}$

$\sqrt{9+16+25}$

$= \sqrt{50}$

$|A| = 5\sqrt{2}$

$|B| = \sqrt{(4)^2+(-1)^2+(6)^2}$

$= \sqrt{16+1+36}$

$|B| = \sqrt{53}$

$|C| = \sqrt{(5)^2+(2)^2+(-3)^2}$

$= \sqrt{25+4+9}$

$|C| = \sqrt{50}$

$\cos\theta = \dfrac{-64}{\sqrt{50}\times\sqrt{53}\times\sqrt{50}} = \dfrac{-64}{364}$

$\cos\theta = 0.1758$

$\theta = \cos^{-1} - 0.1758$

$\theta = 100^\circ$

ii i.) $(A+B+C) = (3i+2j+5k)+(2i-j+6k)+(5i+2j-3k)$

$= 10i+3j+8k$

$|(A+B+C)| = \sqrt{(10)^2+(3)^2+(8)^2}$

$= \sqrt{100+9+64} = \sqrt{173}$

$|(A+B+C)| = \sqrt{173}$

$\therefore$ Unit vector $(\hat{u}) = \dfrac{10i+3j+8k}{\sqrt{173}}$ $\therefore \hat{u} = \dfrac{10j}{\sqrt{173}} + \dfrac{3i}{\sqrt{173}} + \dfrac{8k}{\sqrt{173}}$

3) If $f = 3ui + u^2j + (u+2)k$ and $v = 2ui - 3uj + (u-2)k$, evaluate the integral of $(f \times v)du$ from 0 to 1

$$f \times v = \begin{vmatrix} i & j & k \\ 3u & u^2 & (u+2) \\ 2u & -3u & (u-2) \end{vmatrix}$$

$$= i\begin{vmatrix} u^2 & (u+2) \\ -3u & (u-2) \end{vmatrix} - j\begin{vmatrix} 3u & (u+2 \\ 2u & (u-2 \end{vmatrix} + k\begin{vmatrix} 3u & u^2 \\ 2u & -3u \end{vmatrix}$$

$= i[(u^3-2u^2)-(-3u^2-6u)] - j[(3u^2-6u)-(2u^2+4u)] + k[(-9u^2-2u^3)]$

$= [u^3+u^2+6u]i - [u^2-10u]j + [-9u^2-2u^3]k$

$\int(f \times v) = \left[\frac{u^4}{4} + \frac{u^3}{3} + \frac{6u^2}{2}\right]_i + \left[-\frac{u^3}{3} + \frac{10u^2}{2}\right]_j + \left[-\frac{9u^3}{3} - \frac{2u^4}{4}\right]k$

$\int_0^1 (f \times v)du = \left[\frac{3+4+36}{12}\right]i + \left[\frac{-1+15}{3}\right]j + \left[-\frac{6-1}{2}\right]k + [0]^0$

$\int_0^1 (f \times v)du = (43/12)i + (14/3)j - (7/2)k$

$\int_0^1 (f \times v)du = (3.58i + 4.67j - 3.5k$