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19/ENGDS/DS9

MECHATRONICS

MAT 102

1. $A \Rightarrow 3i + 7j - 2k$

$$B \Rightarrow i + 3j + 7k$$

$$C \Rightarrow 9i - 4j + 6k$$

From the general formula

$$\vec{u} \cdot \vec{v} = |\vec{u}| |\vec{v}| \cos \theta$$

(i) angle between A and C

$$\vec{A} \cdot \vec{C} = |A| |C| \cos \theta$$

$$\begin{aligned} \Rightarrow (3i + 7j - 2k) \cdot (9i - 4j + 6k) &= 27 - 28 - 12 \\ &= -13 \end{aligned}$$

$$|A| = \sqrt{3^2 + 7^2 + (-2)^2} = \sqrt{9 + 49 + 4} = \sqrt{62}$$

$$|C| = \sqrt{9^2 + (-4)^2 + 6^2} = \sqrt{81 + 16 + 36} = \sqrt{133}$$

$$\cos \theta = \frac{\vec{A} \cdot \vec{C}}{|A| |C|}$$

$$= \frac{-13}{\sqrt{62} \sqrt{133}}$$

$$= -0.143$$

$$90.807$$

$$\cos \theta = -0.143$$

$$\theta = \cos^{-1}(-0.143)$$

$$= 98.23^\circ$$

(ii) B and C

$$B \Rightarrow i + 3j + 7k$$

$$C \Rightarrow 9i - 4j + 6k$$

$$\cos \theta = \frac{\vec{B} \cdot \vec{C}}{|\vec{B}| |\vec{C}|}$$

$$\vec{B} \cdot \vec{C} = (i + 3j + 7k) \cdot (9i - 4j + 6k)$$
$$= 9 - 12 + 42$$

$$= 45$$

$$|\vec{B}| \Rightarrow \sqrt{1^2 + 3^2 + 7^2} = \sqrt{1 + 9 + 49} = \sqrt{59}$$

$$|\vec{C}| \Rightarrow \sqrt{9^2 + (-4)^2 + 6^2} = \sqrt{81 + 16 + 36} = \sqrt{133}$$

$$\cos \theta = \frac{45}{\sqrt{59} \cdot \sqrt{133}}$$

$$= 0.508$$

$$= 59.47^\circ$$

$$88.58$$

$$\cos \theta = 0.508$$

$$\theta = \cos^{-1}(0.508)$$

$$= 59.47^\circ$$

$$i) (A+B+C) = (3i + 7j - 2k) + (i + 3j + 7k)$$

$$(A+B) = 4i + 10j - 5k$$

$$(A+B) + C = 4i + 10j - 5k + 9i - 4j + 6k$$
$$= 13i - 6j - k$$

$$|A+B+C| = \frac{\vec{A+B+C}}{|\vec{A+B+C}|}$$

$$\Rightarrow |A+B+C| = \sqrt{13^2 + (-6)^2 + (-1)^2}$$
$$= \sqrt{169 + 36 + 1}$$

$$= \sqrt{206}$$

$$\therefore \vec{A+B+C} = \frac{13i - 6j - k}{\sqrt{206}}$$

$$\sqrt{206}$$

$$2. \quad x = -8t^2 \quad y = t^2 - 4t, \quad z = t - 1$$

$$s = -8t^2 + t^2 - 4t + t - 1$$

$$\text{Velocity} = \frac{ds}{dt} = -28t + 2t - 4 + 1 - 0$$

$$= -16t + 2t - 4 + 1$$

$$\text{Acceleration} = \frac{d^2s}{dt^2} = -16 + 2 - 0 + 0$$

$$= -14 \text{ms}^{-1}$$

$$3. \quad A \Rightarrow 4i + 2j - 4k$$

$$B \Rightarrow 8i - 2j + k$$

$$C \Rightarrow i + 4j - 3k$$

$$(A \times B) \times C$$

$$= A \times B = \begin{matrix} & i & j & k \\ \begin{matrix} 4 & 2 & -4 \\ 8 & -2 & 1 \end{matrix} \end{matrix}$$

$$\begin{vmatrix} 4 & 2 & -4 \\ 8 & -2 & 1 \end{vmatrix}$$

$$i [(2 \times 1) - (-4 \times -2)] - j [(4 \times 1) - (-4 \times 8)] + k [(4 \times -2) - (2 \times 8)]$$

$$i [2 - 8] - j [4 + 32] + k [-8 - 16]$$

$$-6i - 36j - 24k = (A \times B)$$

$$(A \times B) \times C = \begin{matrix} & i & j & k \\ \begin{matrix} -6 & -36 & -24 \\ 1 & 4 & -3 \end{matrix} \end{matrix}$$

$$\begin{vmatrix} -6 & -36 & -24 \\ 1 & 4 & -3 \end{vmatrix}$$

$$i [(36 \times -3) - (-24 \times 4)] - j [(-6 \times -3) - (-24 \times 1)]$$

$$+ k [(-6 \times 4) - (-36 \times 1)]$$

$$= i [108 + 96] - j [18 + 24] + k [-24 + 36]$$

$$i [204] - j [42] + k [12]$$

$$204i - 42j + 12k$$