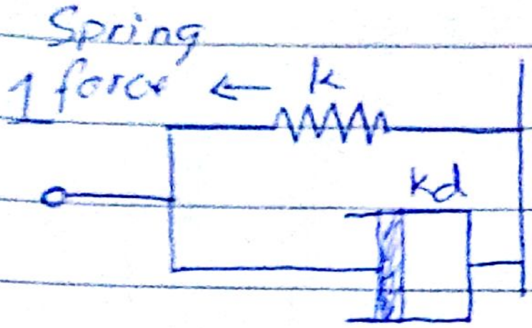
UGWUJA OGBONNA 17/Eng04/072 Electrical

System response I



Spring $\rightarrow k(x-0)$

$F_{damper} = kd \dfrac{\partial(x-0)}{\partial t}$

$F(t) \rightarrow F(t)$

Newton's law $\rightarrow F(t) - k(x-0) - kd\dfrac{d(x-0)}{dt} = 0$

$0 = F(t) - kx - kd\dfrac{dx}{dt} \rightarrow F(s) - kx(s) - kd\,s\,x(s)$

$F(s) - kx(s) - kd\,s\,x(s) = 0$

$F(s) = [k + kds]\,x(s)$

$G(s) = \dfrac{x(s)}{F(s)} = \dfrac{1}{k + kds} = \dfrac{1/k}{1 + [kd/k]s} \rightarrow \dfrac{1/k}{Ts + 1}$

$\Rightarrow T = \dfrac{kd}{k} \Rightarrow \dfrac{0.03}{4 \times 10^3} = 0.75 \times 10^{-5}$

$= 7.5\,\mu s.$

2. Let's assume E_2 - Final Energy

E_1 - initial Energy

$E_2 = mc\Delta\theta \rightarrow mc(\theta_2 - \theta_1)$

$E_1 = mc\Delta\theta \rightarrow mc(\theta_2 - \theta_1$

θ is the new temperature of the metal

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$$G(s) = \frac{E_2}{E_1} = \frac{mc(\theta - \theta_1)}{mc(\theta_2 - \theta_1} = \frac{1}{Ts+1}$$

$$= \frac{\theta - \theta_1}{\theta_2 - \theta_1}(s) = \frac{1}{Ts+1}$$

$$\theta - \theta_1 = \frac{\theta_2 - \theta_1}{Ts+1}$$

Let $\theta_2 - \theta_1 (s) = k(t)$

$$(\theta - \theta_1)(s) = \frac{k(t)}{Ts+1}$$

$$(\theta - \theta)(s) = \frac{k(t)(1/T)}{s + 1/T} = \frac{k}{s}$$

$$Lap^{-1}(k/s) = k(t)$$

2// $$(\theta - \theta_1)(s) = \frac{k(1/T)}{s(s+1/T)}$$

taking the ~~laplace inverse~~ inverse laplace

$$(\theta - \theta_1) = k(1 - e^{-t/T})$$

$$k = \theta_2 - \theta_1$$

$$(\theta - \theta_1) = (\theta_2 - \theta_1)[1 - e^{-t/T}]$$

3 $$\frac{\omega}{kmX} = \frac{1}{Ts+1}$$

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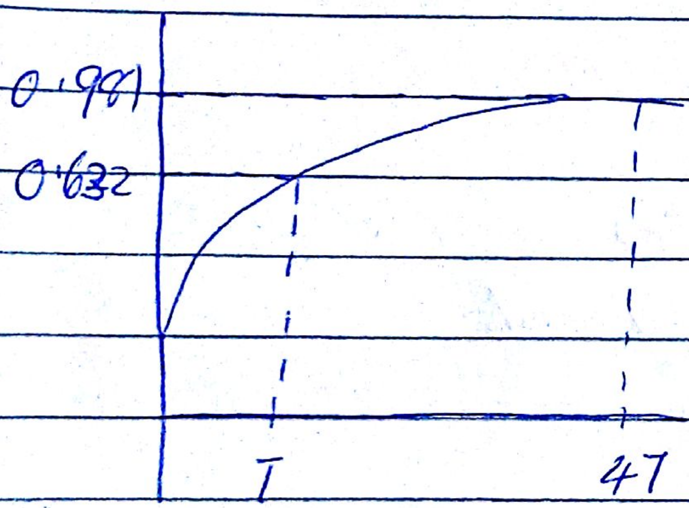
$$T = \frac{1}{K_S} \qquad K_m = \frac{K_1 K_2}{K_3}$$

$$\omega = \frac{K m x}{T s + 1} = \frac{K m x}{s}\left(\frac{1}{T s + 1}\right)$$

$$\frac{K m x}{s}\left(\frac{1/T}{s + 1/T}\right)$$

$$\omega(t) = K m x \left[1 - e^{-t/T}\right.$$



at $t = 0$ $\;Kmx(1 - e^{0}) = 0$

at $t = T$ $\;Kmx(1 - e^{-1t/k}) = 0.632\, kmx$

at $t = 4T$ $\;kmx(1 - e^{-4T/t}) = 0.981\, kmx$

for $t = T$

$$\Delta\% = [0.632 - 0] \times 100\% = 63.2\%$$

for $t = 4T$

$$\Delta\% = [0.981 - 0] \times 100\% = 98.1\%$$

4 $\quad \theta_1(t) = Ct$

$$\theta_1(s) = \frac{C}{s^2}$$

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$$\frac{\theta_o}{\theta_i}(s) = \frac{1}{3s+1}$$

$$\theta_o(s) = \frac{\theta_i(s)}{3s+1}$$

$$\theta_o(s) = \frac{c}{s^2(3s+1)} = \frac{c/3}{s^2(s+1/3)}$$

$$\theta_o(t) = c[(t-3)][1-e^{-t/3}] \quad --- (1)$$

when t is large

$$\theta_o t = c[t-3(1)]$$

$$\theta_o t = ct - 3c$$

$$\theta_e = \theta_i - \theta_o = ct - (ct - 3c) = 3c$$

when $T = 3$, $c = 4$ mm/s

after 2 seconds

$$\theta_i = 4 \times 2 = 8mm$$

$$\theta_e = c \times 3 = 4 \times 3 = 12mm$$

θ_o from (1)

$$\theta_o = 4[2-3(1-e^{-2/3})] = 2.161mm$$

5i

$$\frac{2}{0.2s+0.5} = \frac{2/0.5}{0.2s/0.5+1}$$

$$\frac{4}{0.4s+1} = \frac{k}{\tau s+1}$$

$4 = $ D.C gain $\quad 0.4 = $ time constant

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ii $\frac{0.2}{0.05s + 0.1} = \frac{0.2/0.4}{\frac{0.05s}{0.4} + \frac{0.4}{0.4}}$

$\frac{2}{0.5s+1}$ $\therefore$ 2 = DC gain 0.5 = time constant

iii $\frac{2}{3s+1}$ = 2 = DC gain

3 = time constant

iv $\frac{16}{8s+4} = \frac{16/4}{8/4 s + 1}$

$= \frac{4}{2s+1}$ = 4 = DC gain

2 = time constant

6 $\frac{\omega(s)}{\theta} = \frac{k_m}{T_m s + 2}$

if $k_m = 15s^{-1}$

$T_m = 4s$

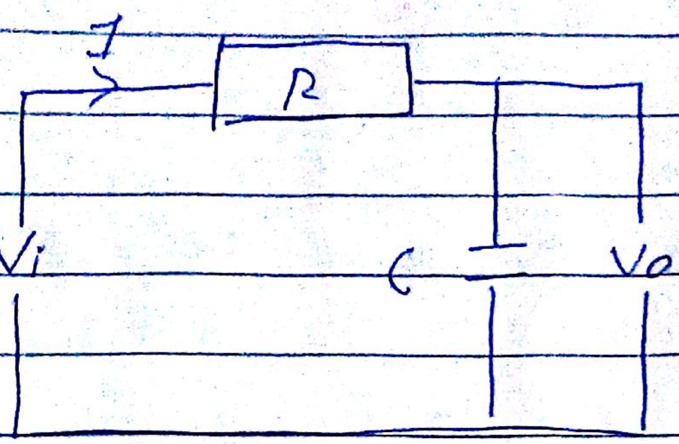
$\frac{15}{4s+2} = \frac{15/2}{4/2 s + 1} = \frac{7.5}{2s+1}$

DC gain $= 7.5s^{-1}$

Time Constant 2

SYSTEM RESPONSE II

1



$$\left(\frac{V_o}{V_i}\right)(s) = \frac{1}{\tau s + 1}$$

where $\tau = RC$

$R = 47\Omega;\ C = 20\mu F$

$V_i = 5\sin(2000t)$

$$G(s) = \frac{1}{\tau s + 1}$$

$$G(j\omega) = \frac{1}{9.4\times10^{-4} j\omega + 1} \times \frac{9.4\times10^{-4} j\omega - 1}{9.4\times10^{-4} j\omega - 1}$$

$$G(j\omega) = \frac{9.4\times10^{-4} j\omega - 1}{(9.4\times10^{-4})^2 \omega^2 - 1}$$

$$G(j\omega = \frac{-1}{(9.4\times10^{-4})^3 \omega^2 - 1} + \frac{9.4\times10^{-4} j\omega}{(9.4\times10^{-4})^2 \omega^2 - 1}$$

$$\cancel{\theta} = \tan^{-1}\left[\frac{\dfrac{9.4\times10^{-4}(2000)}{9.4\times10^{-4}\omega^2 - 1}}{\dfrac{1}{9.4\times10^{-4}\omega^2 - 1}}\right]$$

$$= \tan^{-1}\left[\frac{9.4\times10^{-4}(2000)}{-1}\right]$$

$$\cancel{\theta} = \underline{-61.99^\circ}$$

$$|G(j\omega)| = \frac{1}{\sqrt{(9.4\times10^{-4})\omega^2+1^2}} = \frac{1}{\sqrt{(9.4\times10^{-4})^2(2000)^2+1}}$$

$$= 0.4696$$

V_0 = amplitude $= 5\times0.4696 = 2.348$

$$\theta = 2000 - 61.99^\circ$$

$$V_0 = 2.348\sin(2000t - 61.99^\circ)$$

$$V_0 = 2.35\sin(2000t - 62^\circ)$$

2 $$\frac{x_o}{x_i} = \frac{1}{T^2s^2+2\delta Ts+1}$$

$$G(s) = \frac{1}{(1-T^2s^2)+2\delta Ts}$$

$$G(j\omega = \frac{1}{(1-T^2\omega^2)+2\delta Tj\omega}$$

$\delta = 0.2$ $T = 0.4s$ $\omega = 2.5$ rads

$$G(j\omega = \frac{1-T^2\omega^2 - 2\delta Tj\omega}{(1-T^2\omega^2)^2+4\delta^2T^2\omega^2}$$

$$= \frac{1-(0.4^2)^2(2.5)^2 - 2(0.2)(0.4)(2.5)}{(1-(0.4)^2(2.5)^2)^2+4(0.2)^2(0.4)^2(2.5)^2}$$

$G(j\omega) = 0 + 0.25$

$\phi = \tan^{-1}\left(\frac{2.5}{0}\right) = \infty$

$\tan^{-1}\infty = \underline{90^{\circ}}$

$|G(j\omega)| = \sqrt{0^2 + (2.5)^2}$

$= 2.5$

Amplitude $= 6 \times 2.5 = \underline{\underline{15}}$