MAT102 Assignment [illegible]

$M = pi - 6j - 3k$

$N = 4i + 3j - k$

$O = i - 3j + 2k$

a) for perpendicularity btw M & N ($\bar{m} \cdot \bar{v} = 0$)

$\bar{m} \cdot \bar{v} = (pi - 6j - 3k) \cdot (4i + 3j - k) = 0$

$4p - 18 + 3 = 0$

$4p = 18 - 3$

$4p = 15$

$\therefore p = 15/4 \quad p = 3.75$

b) for co-planar $\bar{m} \cdot (\bar{n} \times \bar{o}) = 0$

$$\begin{vmatrix} p & -6 & -3 \\ 4 & 3 & -1 \\ 1 & -3 & 2 \end{vmatrix} = 0 \quad p\begin{vmatrix} 3 & -1 \\ -3 & 2 \end{vmatrix} - 6\begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix} - 3\begin{vmatrix} 4 & 3 \\ 1 & -3 \end{vmatrix} = 0$$

$p(3\times2 - (-1\times-3)) + 6(4\times2 - (-1\times1)) - 3(4\times-3 - [3\times1])$

$p(6-3) + 6(8+1) - 3(-12-3) = 0$

$3p + 6(9) - 3(-15) = 0$

$3p + 54 + 45 = 0$

$3p = -54 - 45$

$3p = -99$

$\therefore p = -33$

- let $a = 3i + 2j + 4k$

$b = 2i + j + 0k$

$c = 5i + 3j - 3k$

$\therefore$ the magnitude of $a = \sqrt{3^2 + 2^2 + 5^2} = \sqrt{9+4+25} = \sqrt{38}$

$b = \sqrt{2^2 + (-1)^2 + [illegible]} = \sqrt{4+1+[illegible]} = \sqrt{41}$

$c = \sqrt{5^2 + [illegible]} = [illegible] = \sqrt{38}$

- The direction cosine and [illegible] $C =$

$\frac{3}{\sqrt{38}}i + \frac{2}{\sqrt{38}}$ cos a

2) $3i+2j+5k$, $2i-j+6k$ and $5i+2j-3k$

The sum $= 3i+2j+5k+2i-j+6k+5i+2j-3k$

$= 10i+3j+8k$

∴ the unit vector $= \frac{\text{vector}}{\text{magnitude}}$

magnitude $\sqrt{10^2+3^2+8^2} = \sqrt{100+9+64} = \sqrt{173}$

∴ The unit vector $= \frac{10}{\sqrt{173}}i + \frac{3}{\sqrt{173}}j + \frac{8}{\sqrt{173}}k$

Direction cosine: $\cos\alpha = \frac{10}{\sqrt{173}}$

$\cos\beta = \frac{3}{\sqrt{173}}$

$\cos\gamma = \frac{8}{\sqrt{173}}$

3 $F = 3ui+u^2j+(u+2)k$ on c $v = 2ui-3uj+(u-2)k$

$\therefore (F \times v) = 6u^3i - 3u^3j + u^2 - 4k$

$\therefore \int_0^1 (F \times v)\,du = i\left(\frac{6u^3}{3}\right)\Big|_0^1 - j\left(\frac{3u^4}{4}\right)\Big|_0^1 + k\left(\frac{u^3}{3} - \frac{4u}{1}\right)\Big|_0^1$

$= 2u^3i|_0^1 - j\left(\frac{3u^4}{4}\right)\Big|_0^1 + k\left(\frac{u^3}{3} - 4\right)\Big|_0^1$

$i(2^3)|_0^1 - j\left(\frac{3u^4}{4}\right)\Big|_0^1 + k\left(\frac{u^3}{3} - 4\right)\Big|_0^1$

$i(2(1)^3 - 0) - j\left(\frac{3(1)^4}{4} - 0\right) + u\left(\frac{(1)^3}{3} - 4 - 0\right)$

$i(2) - j\left(\frac{3}{4}\right) + \left(-\frac{11}{3}\right)k$

$\therefore \int F \times v = 2i - \frac{3}{4}j - \frac{11}{3}k$