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DEPARTMENT: MECHATRONICS.

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Assignment

1. If $M = pi - 6j - 3k$ $N = 4i + 3j - k$ $O = i - 3j + 2k$ find the value of p for which

a. M & N are perpendicular to each other M & N are coplanar.

b. M & N & O are coplanar.

solution

$M = pi - 6j - 3k$ $N = 4i + 3j - k$ $O = i - 3j + 2k$.

Perpendicular: $\bar{A} \cdot \bar{B} = 0$.

$\therefore \bar{M} \cdot \bar{N} = 0$

$\bar{M} \cdot \bar{N} = (pi - 6j - 3k) \cdot (4i + 3j - k)$

$= 4p - 18 + 3$

$= 4p - 15$

for perpendicular vectors.

$\bar{M} \cdot \bar{N} = 0$.

$0 = 4p - 15$

$0 + 15 = 4p$

$15 = 4p$

$p = \frac{4}{15}$

coplanar:

M, N & O are coplanar.

$\bar{A} \cdot (\bar{B} \times \bar{C}) = 0$.

$\bar{M} \cdot (\bar{N} \times \bar{O}) = 0$

$$\bar{M} \cdot (\bar{N} \times \bar{O}) = \begin{vmatrix} p & -6 & -3 \\ 4 & 3 & -1 \\ 1 & -3 & 2 \end{vmatrix}$$

$$p \begin{vmatrix} 3 & -1 \\ -3 & 2 \end{vmatrix} - (-6) \begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix} + (-3) \begin{vmatrix} 4 & 3 \\ 1 & -3 \end{vmatrix}$$

[illegible] are perpendicular to each other [illegible] find the value of p for which
b. M, N & O are coplanar.

<u>Solution</u>

$M = pi - 6j - 3k \quad N = 4i + 3j - k \quad O = i - 3j + 2k.$

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$p(6 - 3) + 6(8 - (-1)) - 3(-12 - 3)$

$p(3) + 6(8 + 1) - 3(-15)$

$3p + 6(9) + 45$

$3p + 54 + 45$

$= 3p + 99$

$\bar{A} \cdot (\bar{B} \times \bar{C}) = 0.$

$0 = 3p + 99$

$0 - 3p = 99$

$\frac{-3p}{3} = \frac{99}{3} \quad -p = 33 \quad p = -33$

2. Find the direction cosines and the unit vector along the sum of $3i+2j+5k$, $2i-j+6k$ and $5i+2j-3k$.

solution

$3i+2j+5k, 2i-j+6k$ and $5i+2j-3k$

let the sum be R

$(3i+2j+5k)+(2i-j+6k)+(5i+2j-3k)$

$(3+2+5)i+(2-1+2)j+(5+6+(-3))k$

$= 10i+(1+2)j+(8k)$

$= 10i+3j+8k$

Direction cosine $R = 10i+3j+8k$.

$|R| = 10i+3j+8k$

$a_x = 10 \quad a_y = 3 \quad a_z = 8$

$|R| = \sqrt{10^2+3^2+8^2}$

$|R| = \sqrt{100+9+64}$

$= \sqrt{173}$.

$\cos\alpha = \frac{10}{\sqrt{173}} = 0.7603$

$\cos\beta = \frac{3}{\sqrt{173}} = 0.2281$

$\cos\gamma = \frac{8}{\sqrt{173}} = 0.6082$

$\sqrt{178}$ $=$

3. If $f = 3ui + u^2 j + (u+2)k$ $v = 2ui - 3uj + (u-2)k$ evaluate the integral of $(f \times v)\,du$ from 0 to 1

Solution

$f = 3ui + u^2 j + (u+2)k$ $\quad v = 2ui - 3uj + (u-2)k$

$$f \times v = \begin{vmatrix} i & j & k \\ 3u & u^2 & u+2 \\ 2u & -3u & u-2 \end{vmatrix}$$

$$i\begin{vmatrix} u^2 & u+2 \\ -3u & u-2 \end{vmatrix} - j\begin{vmatrix} 3u & u+2 \\ 2u & u-2 \end{vmatrix} + k\begin{vmatrix} 3u & u^2 \\ 2u & -3u \end{vmatrix}$$

$i(u^2(u-2) - (-3u(u+2)) - j(3u(u-2) - (2u(u+2)) + k(3u(-3u) - u^2(2u))$

$i(u^3 - 2u^2 - (-3u^2 - 6u) - j(3u^2 - 6u - 2u^2 + 4u) + k(-9u^2 - 2u^3)$

$i(u^3 - 2u^2 + 3u^2 + 6u) - j(3u^2 - 2u^2 - 6u + 4u) + k(-9u^2 - 2u^3)$

$i(u^3 + u^2 + 6u) - j(u^2 - 2u) + k(-9u^2 - 2u^3)$

$(u^3 + u^2 + 6u)i - (u^2 - 2u)j + (-9u^2 - 2u^3)k.$

$$(f \times v)\,du = \left[(u^3 + u^2 - 6u^{\circ}\right]i - (u^2 - 2u)j + (-9u^2 - 2u^3)k$$

$$= \left[\left[\frac{u^4}{4} + \frac{u^3}{3} - \frac{6u^2}{2}\right]\right]i - \left[\frac{u^3}{3} - \frac{2u^2}{2}\right]j + \left[-\frac{9u^3}{3} - \frac{2u^4}{4}\right]k\Bigg]$$

$$= \left[\left[\frac{u^4}{4} + \frac{u^3}{3} - 3u^2\right]i - \left[\frac{u^3}{3} - u^2\right]j + \left[-3u^3 - \frac{u^4}{2}\right]k\right]_0^1$$

$$= \left[\left[\frac{1}{4} + \frac{1}{3} - 3(1)^2\right]i - \left[\frac{(1)^3}{3} - 1^2\right]j + \left[-3(1)^2 - \frac{(1)^4}{2}\right]k\right]^1$$

$$= \left[\frac{1}{4} + \frac{1}{3} - 3\right]i - \left[\frac{1}{3} - 1\right]j + \left[-3 - \frac{1}{2}\right]k$$

$$= \frac{-29}{12}i - \left[-\frac{2}{3}\right]j + \left[-\frac{7}{2}\right]k$$

$$= \frac{-29}{12}i + \frac{2}{3}j - \frac{7}{2}k$$