(1) from the diagram

Spring $= lk(x-0)$

$+$ clamper $= lkd\frac{d(x-0)}{dt}$

$f(t) \Rightarrow f(t)$

$$f(t) - lk(x-0) - lkd\frac{d(x-0)}{dt}$$

$$0 = f(t) - lkx - lkd\frac{dx}{dt}$$

Taking the laplace transform

$$\therefore f(s) = lkx(s) - lkds\,x(s) = 0$$

$$f(s) = (lk + lkds)\,x(s)$$

$$G(s) = \frac{x(s)}{f(s)} = 1/lk + lkds$$

$$\Rightarrow \frac{1/lk}{1 + [lkd/lk]s}$$

(2) $\mathcal{E}_2 =$ new energy
$\mathcal{E}_1 =$ Initial energy

$\mathcal{E}_2 = mc\Delta\theta \Rightarrow \mathcal{E}_2 = mc(\theta - \theta_1)$
$\mathcal{E}_1 = mc\Delta\theta \Rightarrow \mathcal{E}_2 = mc(\theta_2 - \theta_1)$

where θ is the new temp of the metal

$$G(s) = \mathcal{E}_2/\mathcal{E}_1 = \frac{mc(\theta - \theta_1)}{mc(\theta_2 - \theta_1)} = 1/_{Ts+1}$$

$$\Rightarrow \frac{\theta - \theta_1(s)}{\theta - \theta_1} = 1/_{Ts+1}$$

let $\theta_2 - \theta_1(s) = 1h(t)$

$$(\theta - \theta_2)(s) = \frac{1h(t)}{Ts+1}$$

$$(\theta - \theta_1)(s) = 1h(t)/_{Ts+1}$$

$$(\theta - \theta_1)(s) = 1h(t)(k)$$

$$s + 1/_T$$

taking the laplace transform of $h(t)$

$$(\theta - \theta_1)(s) = \frac{h\,(1/T)}{s\,(s+1/T)}$$

Inverse laplace transform

$$(\theta - \theta_1) = h\,[1 - e^{-1/T})$$

from (1) = $h = \theta_2 - \theta_1$

$$(\theta - \theta_1) = (\theta_2 - \theta_1)\,[1 - e^{-t/T})$$

(3) $w/_{kmax}$ $\quad 1/_{Tct1}$

$T = 1/_{khs}$ $\quad kmm = k_1 k_2/_{khs}$

$$w = \frac{kmax}{kt+1}$$

laplace transform of the step input

$$w = \frac{kmax}{s}\left(1/_{Ts+1}\right)$$

$$\frac{kmax}{s} \cdot \frac{(1/T)}{(s+1/T)}$$

$ws = kmax\,(1 - e^{-t/T})$

at $t=0$, $kmax\,(1-e^0)$ = initial

at $t=T$, $kmax(1-e^{-t/T}) = 0.632\,kmax$

at $t = 4T \Rightarrow kmax\,(1-e^{-4t/T}) = 0.98\,kmax$

for $t=T$

$\Delta\% = (0.632 - 0)\times 100\% = 63.2\%$

for $t = 4T$

$\Delta\% = (0.981-0)\times 100\% = 98.1\%$