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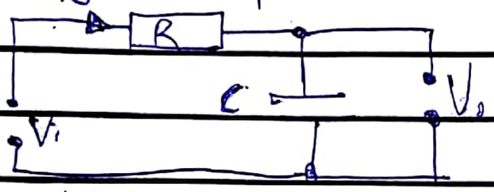
17/Eng04/041

Electrical/Electronics Engineering

Linear Systems

System response II ac



$R = 47\Omega \quad C = 20\mu f$

$$R \quad \frac{V_o}{V_i} = \frac{1}{Ts+T} \qquad T = RC = 47 \times 20 \times 10^{-6} = 940$$

$$= 9.4 \times 10^{-4}$$

$$G(s) = \frac{1}{9.4\times10^{-4} j\omega + 1}$$

$$G(j\omega) = \frac{9.4\times10^{-4} j\omega - 1}{(9.4\times10^{-4})\omega^2 - 1}$$

$$G(j\omega) = \frac{-1}{(9.4\times10^{-4})^2\omega^2 - 1} + \frac{9.4\times10^{-4} j\omega}{(9.4\times10^{-4})^2\omega^2 - 1}$$

$$\phi = \tan^{-1}\left[\frac{\left(\frac{9.4\times10^{-4}(2000)}{(9.4\times10^{-4})\omega}\right)}{\left(\frac{-1}{(9.4\times10^{-4})\omega^2 - 1}\right)}\right] = 1$$

$$\Rightarrow \tan^{-1}\left(\frac{9.4\times10^{-4}(2000)}{-1}\right)$$

$$\phi = -61.99$$

$$|G(\omega)| = \frac{1}{\sqrt{(9.4\times10^{-4})^2\omega^2 + 1^2}} = \frac{1}{\sqrt{(9.4\times10^{-4})^2(2000)^2 + 1}}$$

$$\Rightarrow 0.4696$$

V_o amplitude $= 5 \times 0.4696 = 2.348$

$\phi = \cancel{20000}\ 2000t - 62.99$

V_o = amplitude = $5 * 0.4696 = 2.348$

$\phi = 2000t - 67.99$.

$V_o = 2.348 \sin(2000t - 67.99^\circ)$

$V_o = 2.348 \sin(200t - 67.99^\circ)$

2) $\frac{x_o}{x_i} = \frac{1}{T s^2 + 2\delta s + 1} = G(s) = \frac{1}{(1 - T^2 s^2) + 2\delta s}$

$G(j\omega) = \frac{1}{(1 - T^2\omega^2) + 2\delta T j\omega}$

$\delta = 0.2 \quad T = 0.4 sec \quad \omega = 2.5 rad$

$G(j\omega) = \frac{1 - T^2\omega^2 - 2\delta T j\omega}{(1 - T^2\omega^2)^2 + 4\delta T^2 \omega^2}$

$= \frac{1 - (0.4)^2(2.5)^2 - 2(0.2)(0.4)(2.5)}{(1 - (0.4)^2(2.5)^2)^2 + 4(0.2)^2(0.4)^2(2.5)^2}$.

$G(s) = 0 - 2.5$

$\phi = \tan^{-1}\left(\frac{2.5}{0}\right) = \tan^{-1}(\infty) = 90$

$|G j\omega| = \sqrt{0^2 + (-2.5^2)}$

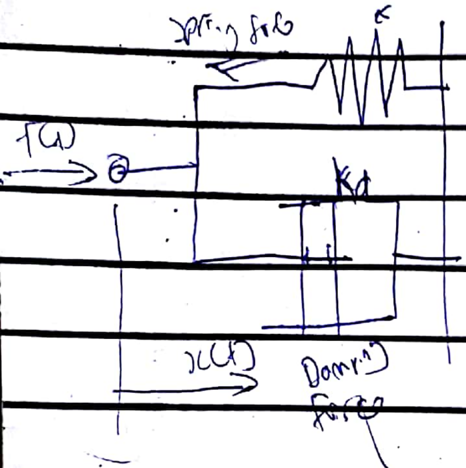
$= 2.5$

amplitude $= G * \theta \Rightarrow 6 \times 2.5 = 15$

TAModel.m

System response I

Show derivation of TF of spring & damper and $k_d = 0.03$ $k = 4kN/m$



spring $\Rightarrow k(x-0)$

$f_{damper} = k_d \frac{d(x-0)}{dt}$

$f(t) \Rightarrow f(t)$

Newton law $\Rightarrow f(t) - k(x-0) - k_d \frac{d(x-0)}{dt} = 0$

$0 = f(t) - k(x) - k_d \frac{dx}{dt} \Rightarrow$ laplace $= f(s) - kx(s) - k_d s\, x(s)$

$f(s) - kx(s) - k_d s\, x(s) = 0$

$f(s) = (k + k_d s)\, x(s)$

$G(s) = \frac{x(s)}{F(s)} = \frac{1}{k + k_d s} \Rightarrow \frac{1/k}{1 + (k_d/k)s}$

$G(s) = \frac{1/k}{(k_d/k)s + 1} = \frac{1/k}{Ts + 1}$

hence

$T = \frac{k_d}{k} \Rightarrow \frac{0.03}{4\times10^{3}} = 0.75\times10^{-5}$ sec

$T = 7.5\times10^{-6}$ s

$= 7.5\,\mu s$

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4) $G(s) = 1/(3s+1)$

$\theta_i(t) = Ct$

$\theta_i(s) = C/s^2$

$$\frac{\theta_o(s)}{\theta_i} = \frac{1}{3s+1}$$

$$\theta_o(s) = \frac{\theta_i C s}{3s+1} \qquad \theta_o(s) = \frac{C}{s^2(3s+1)}$$

$$\theta_o(t) = \frac{C/3}{s^2(s+1/3)}$$

$\theta_o(t) = C(t - 3(1 - e^{-t/3}))$ — 1

When t is large

$\theta_o(t) = C(t - 3(1))$

$\theta_o t = ct - 3c$

$\theta_e = \theta_i - \theta_o = Ct - (Ct - 3C) = 3C$

$T = 3 \qquad C = 4\,mm/s$

aft 2 secs

$\theta_i = 4 \times 2 = 8\,mm$

$\theta_e = C \times 3 = 4 \times 3 = 12\,mm$ (steady state)

θ_o from 1

$\theta_o = 4(2 - 3(1 - e^{-2/3})) = 2.161005428\,mm$

5) i) $$G(s) = \frac{2}{0.2s + 0.5} = \frac{2/0.5}{0.2s/0.5 + 1}$$

$$= \frac{4}{0.4s + 1} \qquad \text{relate} \quad \frac{K}{Ts + 1}$$

$4 = K \qquad T = 0.4$

DC gain $\qquad$ Time Constant

ii) $\frac{0.2}{0.05s+0.1} = \frac{0.2/0.1}{0.05s/0.1 + 0.1/0.1} = \frac{2}{0.5s+1}$

$\frac{2}{0.5s+0.1}$ $\frac{K}{Ts+1}$

$T = 0.5$ $K = 2$

Time constant DC gain

iii) $\frac{2}{3s+1} = \frac{2/1}{3s/1 + 1/1} = \frac{2}{3s+1}$ $\frac{K}{Ts+1}$

$K = 2$ $T = 3$

DC gain Time constant

iv) $\frac{16}{8s+4} = \frac{16/4}{8s/4 + 4} = \frac{4}{2s+1}$ $\frac{K}{Ts+1}$

$T = 2$ $4 = K$

Time constant DC gain

6) Output speed : W rad/s

$\frac{W(s)}{\theta} = \frac{K_m}{T_m s + 2}$

$K_m = 15s^{-1}$

$T_m = 4$

$= \frac{15}{4s+2} \Rightarrow \frac{15/2}{4s/2 + 2/2} = \frac{7.5}{2s+2}$

$\frac{7.5}{2s+2}$ $\frac{K}{Ts+2}$

$K = 7.5$

DC gain

2) $m = 0.5kg$ $\theta_1 = 20°C$

$c = 346 J/kg K$

$\mathcal{E}_2 = mc\Delta\theta$

$\mathcal{E}_1 = mc\Delta\theta$

$$G(s) = \frac{\mathcal{E}_2}{\mathcal{E}_1} = \frac{mc\Delta\theta_1}{mc\Delta\theta_1}$$

$$\therefore \frac{\Delta\theta}{\Delta\theta} = \frac{\theta - \theta_1}{\theta_2 - \theta_1} = \frac{T}{Ts+1}$$

$$\therefore (Ts+1)(\theta - \theta_1) = \theta_2 - \theta_1$$

$$\theta - \theta_1 = \frac{\theta_2 - \theta_1}{Ts+1}$$

assume $\theta_2 - \theta_1 = K(t)$

$$\theta - \theta_1 = \frac{K(t)}{Ts+1}$$

$$(\theta - \theta_1)(s) = \frac{K(1/T)}{s + 1/T} = \frac{K(1/T)}{S + 1/T}$$

$$\cancel{\theta - \theta_1 = \frac{K(1/T)}{S(s+1/T)}}$$

laplac of $K(t) = K/s$

$$\therefore \theta - \theta_1 = \frac{K}{S} \times \frac{1/T}{(s+1/T)}$$

$$= \frac{K/T}{s(s+1/T)}$$

inverse laplac

$$(\theta - \theta_1) = K(1 - e^{-t/T})$$

$$\therefore \theta = \theta_1 + K(1 - e^{-t/T})$$

recall $K = \theta_2 - \theta_1 =$

$$\theta = \theta_1 + (\theta_2 - \theta_1)(1 - e^{-t/T})$$