i) from the diagram

$Spring = k(x - 0)$

$f_{damper} = kd \frac{d(x-0)}{dt}$

$F(t) \Rightarrow f(t)$

$F(t) - k(x-0) - kd \frac{d(x-0)}{dt}$

$0 = f(t) - kx - kd \frac{dx}{dt}$

Taking the laplace transform

$\therefore f(s) = kx(s) - kds\,x(s) = 0$

$f(s) = (k + kds)\,x(s)$

$G(s) = \frac{x(s)}{f(s)} = \frac{1}{k + kds}$

$\Rightarrow \frac{1/k}{1 + [kd/k]\,s}$

ii)

2) E_2 = new energy

E_1 = initial energy

$E_2 = mc\Delta\theta \Rightarrow E_2 = mc(\theta - \theta_1)$

$E_1 = mc\Delta\theta \Rightarrow E_2 = mc(\theta_2 - \theta_1)$

where θ is the new temp of the metal

$$G(s) = \frac{E_2}{E_1} = \frac{mc(\theta - \theta_1)}{mc(\theta_2 - \theta_1)} = \frac{1}{Ts+1}$$

$$\Rightarrow \frac{\theta - \theta_1(s)}{\theta_2 - \theta_1} = \frac{1}{Ts+1}$$

$$\theta - \theta_1 = \frac{\theta_2 - \theta_1}{Ts+1}$$

let $\theta_2 - \theta_1(s) = k(t)$

$$(\theta - \theta_1)(s) = \frac{k(t)}{Ts+1}$$

$$(\theta - \theta_1)(s) = \frac{k(t)(1/T)}{s + 1/T}$$

taking the laplace transform of $k(t)$

$$= \frac{k}{s}$$

$$(\theta - \theta_1)(s) = \frac{k(1/T)}{s(s+1/T)}$$

3)
$$\frac{w}{k_m x} = \frac{1}{T_s + 1}$$

$T = 1/k_s$ $k_m = \frac{k_1 k_2}{k_s}$

$$w = \frac{k_m x}{T_s + 1}$$

laplace Transform of the step input

$$w = \frac{k_m x}{s}\left(\frac{1}{T_s+1}\right)$$

$$\frac{k_m x}{s}\left(\frac{1/T}{s + 1/T}\right)$$

$w(s) \Rightarrow k_m x (1 - e^{-t/T})$

at $t = 0$ $k_m x (1 - e^{0}) = $ Initial

at $t = T$ $k_m x (1 - e^{-T/T}) = 0.63\ k_m x$

at $t = 4T = k_m x (1 - e^{-4T/T})$

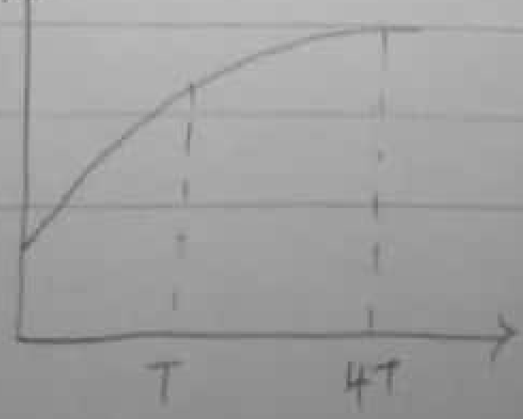
$= 0.981\ k_m x$

for $t = T$

$\Delta\% = (0.632 - 0)\times 100\% = 63.2\%$

$t = 4T$

$\Delta\% = (0.981 - 0)\times 100\% = 98.1\%$



Inverse laplace transform

$$(\theta - \theta_1) = K\left[1 - e^{-t/\tau}\right]$$

from (1) $K = \theta_2 - \theta_1$

$$(\theta - \theta) = (\theta_2 - \theta_1)\left[1 - e^{-t/\tau}\right]$$