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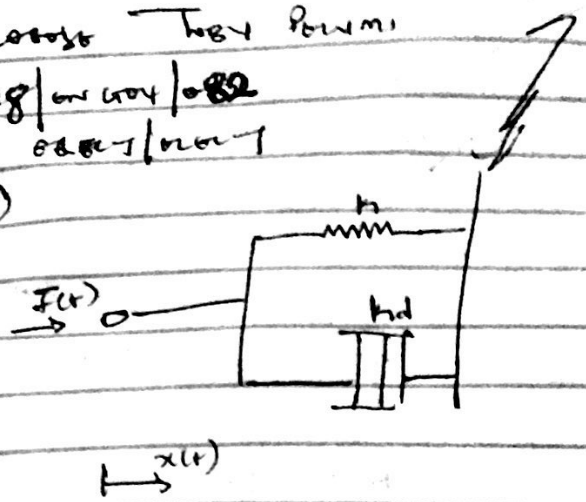
18/ENG04/082

ELECT/ELECT

(1)



$F_{spring} = kx$

$F_{damper} = k_d \, dx/dt$

$F(t) = kx + k_d \, dx/dt$

using laplace transform

$F(s) = kx(s) + k_d s \, x(s)$

$F(s) = [k + k_d s] x(s)$ —— (1)

$$G(s) = \frac{x(s)}{F(s)} = \frac{1/k}{Ts + 1} \quad — (2)$$

also

$$G(s) = \frac{x(s)}{F(s)} = \frac{1}{k + k_d s} \quad — (3)$$

$F(s) + 1 \Leftrightarrow k_d s + k$

$Ts + 1 \Leftrightarrow 1 + k_d s/k$

$$T\cancel{s} = \frac{k_d \cancel{s}}{k}$$

$$= \frac{k_d}{k} = \frac{0.05}{4m} = 7.5 \times 10^{-6}$$

$= 7.5 \mu s$

2) Energy: $mc\Delta\theta$

$E_2 = mc(\theta - \theta_1)$

$E_1 = mc(\theta_2 - \theta)$

$$G(s) = \frac{E_2}{E_1} = \frac{mc(\theta-\theta)}{mc(\theta_2-\theta)} = \frac{1}{Ts+1}$$

$$\theta - \theta_1 = \frac{\theta_2 - \theta_1}{Ts+1}$$

Let $\theta_2 - \theta_1 = k(t)$

$$\theta - \theta_1 = \frac{k(s)}{Ts+1} = \frac{k(1/T)}{s + 1/T}$$

taking laplace transform $k(s) = k/t$

$$\theta - \theta_1 = \frac{k(1/T)}{s(s+1/T)}$$

inverse laplace

$$\theta - \theta_1 = k(1 - e^{-t/T})$$

recall that $k = \theta_2 - \theta$

$$\theta - \theta_1 = \theta_2 - \theta_1 (1 - e^{-t/T})$$

$$\theta = (\theta_2 - \theta_1)(1 - e^{-t/T}) + \theta_1$$

$$119 = (120 - 20)(1 - e^{-360/T})$$

$$99 = 100 - 100e^{-360/T}$$

$$0.01 = e^{-360/T}$$

$$\ln 0.01 = -360/T$$

$$T = 78.17s.$$

Thermal resistance $= T/C$

$C = mc = 0.5 \times 346 = 173 J/K$

$$\therefore R = \frac{78.17}{173} = 0.452 \, K/W$$

$\frac{W}{k_m x} = \frac{1}{Ts+1}$

$k_m = \frac{k_n}{k_s}$

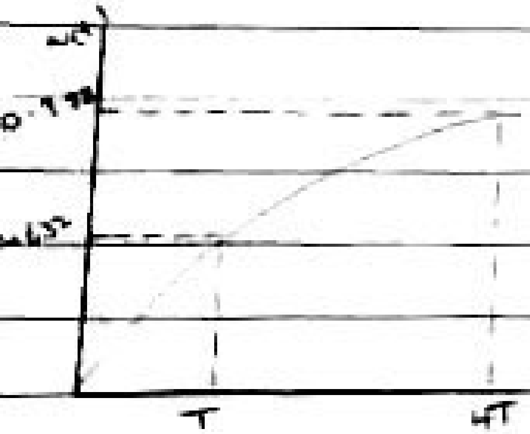
$W = \frac{k_m x}{Ts+1}$

Taking Laplace transform of step input

$W = \frac{k_m x}{s}\left[\frac{1}{Ts+1}\right]$

$= \frac{k_m x}{s}\left[\frac{1/T}{s+1/T}\right]$

$W(t) = k_m x\{1 - e^{-t/T}\}$



at $t = 0$, $k_m x(1 - e^{-t/T}) = 0$

at $t = T$, $k_m x(1 - e^{-T/T}) = 0.632\, k_m x$

at $t = 4T$, $k_m x(1 - e^{-4T/T}) = 0.982\, k_m x$

For $t = T$,

$\Delta\% = (0.632 - 0) \times 100 = \underline{\underline{63.2\%}}$

For $t = 4T$

$\Delta\% = (0.982 - 0) \times 100 = \underline{\underline{98.2\%}}$

Scanned with CamScanner

4) $G(s) = \frac{\theta_o}{\theta_i} = \frac{1}{3s+1}$

$\theta_o(s) = \frac{\theta_i(s)}{3s+1}$

$\theta_o(s) = \frac{C}{s^2(3s+1)} = \frac{C/3}{s^2(s+1/3)}$

$\frac{C/3}{s^2(s+1/3)} = \frac{As+B}{s^2} + \frac{C}{s+1/3}$

$C/3 = (As+B)(s+1/3) + Cs^2$

$\theta_o(t) = c[t - 3(1-e^{-t/3})]$ ...... ①

at $t = \infty$

$\theta_o(t) = c[t - 3(1)]$

$= ct - 3c$

$\theta_e = \theta_i - \theta_o = ct - ct + 3c = 3c$

$T = 3$ $c = 4\,mm/s$

after 2 secs

$\theta_i = 4 \times 2 = 8\,mm$

$\theta_e = c \times 3 = 4 \times 3 = 12\,mm$

θ_o at ①

$\theta_o = 4[2 - 3(1 - e^{-2/3})] = 2.161$

(ii) $G(s) = \dfrac{2}{0.2s + 0.5} = \dfrac{2/0.5}{\frac{0.2}{0.5}s + 1}$

compared to $\dfrac{k}{Ts + 1}$

DC gain $= 2/0.5 = 4$ Time const $= T = 0.2/0.5 = 0.4$

ii) $\dfrac{0.2}{0.05s + 0.1} = G(s) = \dfrac{0.2/0.1}{\frac{0.05}{0.1}s + 1}$

DC gain $= 0.2/0.1 = 2$ Time const $= 0.05/0.1 = 0.5$

(iii) $G(s) = \dfrac{2}{5s + 1}$

DC gain $= 2$ Time const $= 5$

(iv) $G(s) = \dfrac{16}{8s + 4} = \dfrac{16/4}{\frac{8}{4}s + 1}$

DC gain $= 16/4 = 4$ Time const $= 8/4 = 2$

6) $\dfrac{\omega(s)}{\theta} = \dfrac{k_m}{T_m s + 2}$ if $k_m = 15$ $T_m = 4$

$= \dfrac{15}{4s + 2} = \dfrac{15/2}{\frac{4}{2}s + 1}$

DC gain $= 15/2 = 7.5$ Time const $= 2$ s.