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Question 1

Let $A = 4i + j - 2k$; $B = 3i - 2j + k$ and $C = i - 2k$. Find;

a $(A - 2B) \times C$

b $A \times (2C \times 3B)$

Solution

a $A = 4i + j - 2k$

$$2B = 2(3i - 2j + k) = 6i - 4j + 2k$$

$$C = i - 2k$$

$$A - 2B = (4i + j - 2k) - (6i - 4j + 2k)$$

$$A - 2B = 4i - 6i + j + 4j - 2k - 2k$$

$$A - 2B = -2i + 5j - 4k$$

$$(A - 2B) \times C = \begin{vmatrix} i & j & k \\ -2 & 5 & -4 \\ 1 & 0 & -2 \end{vmatrix}$$

$$= \begin{vmatrix} 5 & -4 \\ 0 & -2 \end{vmatrix} i - \begin{vmatrix} -2 & -4 \\ 1 & -2 \end{vmatrix} j + \begin{vmatrix} -2 & 5 \\ 1 & 0 \end{vmatrix} k$$

$$= i(-10 - 0) - j(4 - (-4)) + k(0 - 5)$$

$$= -10i - 8j - 5k$$

$$\therefore (A - 2B) \times C = -10i - 8j - 5k$$

$$b) A = 4i + j - 2k$$

$$2C = 2(i - 2k) = 2i - 4k$$

$$3B = 3(3i - 2j + k) = 9i - 6j + 3k$$

$$2C \times 3B = \begin{vmatrix} i & j & k \\ 2 & 0 & -4 \\ 9 & -6 & 3 \end{vmatrix}$$

$$i \begin{vmatrix} 0 & -4 \\ -6 & 3 \end{vmatrix} - j \begin{vmatrix} 2 & -4 \\ 9 & 3 \end{vmatrix} + k \begin{vmatrix} 2 & 0 \\ 9 & -6 \end{vmatrix}$$

$$i(0 - 24) - j(6 - (-36)) + k(-12 - 0)$$
$$= -24i - 42j - 12k$$

$$\therefore 2C \times 3B = -24i - 42j - 12k$$

$$A \times (2C \times 3B) = \begin{vmatrix} i & j & k \\ 4 & 1 & -2 \\ -24 & -42 & -12 \end{vmatrix}$$

$$i \begin{vmatrix} 1 & -2 \\ -42 & -12 \end{vmatrix} - j \begin{vmatrix} 4 & -2 \\ -24 & -12 \end{vmatrix} + k \begin{vmatrix} 4 & 1 \\ -24 & -42 \end{vmatrix}$$

$$i(-12 - 84) - j(-48 - 48) + k(-168 + 24)$$

$$= -96i + 96j - 144k$$

$$\therefore A \times (2C \times 3B) = -96i + 96j - 144k$$

Question 2

$A = Pi - 6j - 3k$; $B = 4i + 3j - k$ and $C = i - 3j + 2k$.
Find the value of P for which A, B and C are
co-planar.

Solution

$$A \cdot (B \times C) = 0$$

$$\therefore A \cdot (B \times C) = \begin{vmatrix} P & -6 & -3 \\ 4 & 3 & -1 \\ 1 & -3 & 2 \end{vmatrix} = 0$$

$$P \begin{vmatrix} 3 & -1 \\ -3 & 2 \end{vmatrix} + 6 \begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix} - 3 \begin{vmatrix} 4 & 3 \\ 1 & -3 \end{vmatrix} = 0$$

$$P(6 - 3) + 6(8 + 1) - 3(-12 - 3) = 0$$

$$3P + 54 + 45 = 0$$

$$3P = -54 - 45$$

$$3P = -99$$

$$P = \frac{-99}{3}$$

3

$$\therefore P = -33$$