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19/SC181001

ARCHITECTURE

If  $M = pi - 6j - 3k$ ,  $N = 4i + 3k - k$ ,  $O = i - 3j + 2k$  Find the value of  $p$  for which (a)  $M$  and  $N$  are perpendicular to each other

b)  $M$ ,  $N$  and  $O$  are coplanar

a)  $M$  and  $N$  are perpendicular to each other  $M \cdot N = (pi - 6j - 3k) \cdot (4i + 3j - k)$

$$= 4p - 18 + 3$$

$$= 4p - 15$$

since they are perpendicular

$$4p - 15 = 0$$

$$\frac{4p}{4} = \frac{15}{4}$$

$$p = \frac{15}{4}$$

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24-24  
4

Date: .....

b) M, N and D are coplanar

$$M \cdot (CN \times OD) = \begin{vmatrix} p & -6 & -3 \\ 4 & 3 & -1 \\ 1 & -3 & 2 \end{vmatrix}$$

$$= p \begin{vmatrix} 8 & -1 & +6 \\ -3 & 2 & 1 \end{vmatrix} + 6 \begin{vmatrix} 4 & -1 & -3 \\ 4 & -1 & 3 \end{vmatrix} - 3 \begin{vmatrix} 4 & 3 \\ 1 & -3 \end{vmatrix}$$

$$= p(8-3) + 6(8+1) - 3(-12-3)$$

$$= 3p + 6(9) - 3(-15) = 0$$

$$= 3p + 54 + 45 = 0$$

$$= 3p + 99 = 0$$

$$3p = -99$$

$$p = -33$$

Find the direction cosines and the unit vector along the sum of  $3i + 2j + 5k$ ,  $2i - j + 6k$  and  $5i + 2j - 3k$

$$\text{Sum} = 3i + 2j + 5k + 2i - j + 6k + 5i + 2j - 3k \\ = (10i + 4j + 8k)$$

$$\vec{r} = \hat{n} = \frac{\vec{a}}{|\vec{a}|}$$

$$\text{Unit vector of } \vec{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{1}{\sqrt{100}}$$

Question

$$\vec{r} = (10i + 4j + 8k) = \frac{1}{\sqrt{100}}$$

$$a_x = 10, a_y = 3 \text{ and } a_z = 8$$

$$|\vec{a}| = \sqrt{10^2 + 3^2 + 8^2} \\ = \sqrt{100 + 9 + 64} \\ = \sqrt{173} = 13.15$$

1/14/2015  
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224

Date: .....

i The direction cosines are

$$\cos \alpha = \frac{a_{11}}{|V|} = \frac{10}{13.15} = 0.761$$

$$\cos \beta = \frac{a_{12}}{|V|} = \frac{3}{13.15} = 0.228$$

$$\cos \gamma = \frac{a_{13}}{|V|} = \frac{8}{13.15} = 0.608$$

ii unit vector

$$e_1 = \frac{V}{|V|} = \frac{10i + 3j + 8k}{13.15}$$

$$F = \bar{F} = 3ui + u^2j + (u+2)k$$

$$V = 2ui + 3uj + (u-2)k$$

$$(F \times V) = \begin{vmatrix} i & j & k \\ 3u & u^2 & (u+2) \\ 2u & 3u & (u-2) \end{vmatrix}$$

$$= i \begin{vmatrix} u^2 & (u+2) \\ 3u & (u-2) \end{vmatrix} - j \begin{vmatrix} 3u & (u+2) \\ 2u & (u-2) \end{vmatrix} + k \begin{vmatrix} 3u & u^2 \\ 2u & 3u \end{vmatrix}$$

$$= i [u^2(u-2) - 3u(u+2)] - j [3u(u-2) - 2u(u+2)] + k [3u \cdot 3u - 2u \cdot u^2]$$

$$= i [u^3 - 2u^2 - 3u^2 - 6u] - j [3u^2 - 6u - 2u^2 - 4u] + k [9u^2 - 2u^3]$$

$$\therefore \int (F \times V) = \int i [u^3 - 5u^2 - 6u] - j [u^2 - 10u] + k [-2u^3 + 9u^2]$$

$$= i \left[ \frac{u^4}{4} - \frac{5u^3}{3} - 3u^2 \right] - j \left[ \frac{u^3}{3} - 5u^2 \right] + k \left[ -\frac{2u^4}{4} + \frac{9u^3}{3} \right] + C$$

$$= \left[ \frac{u^4}{4} - \frac{5u^3}{3} - 3u^2 \right] - j \left[ \frac{u^3}{3} - 5u^2 \right] + k \left[ -\frac{u^4}{2} + 3u^3 \right] + C$$

$$= \left[ \frac{u^4}{4} - \frac{5u^3}{3} - 3u^2 \right] - j \left[ \frac{u^3}{3} - 5u^2 \right] + k \left[ -\frac{u^4}{2} + 3u^3 \right] + C$$

Date: .....

$$\int_0^1 (f \times v) = i \left[ \frac{1}{4} + \frac{1}{3} + 3 \right] - j \left[ \frac{1}{3} - 5 \right] + k \left[ \frac{-1}{2} - 9 \right] + 4 - 4$$

$$\int_0^1 (f \times v) = i \left[ \frac{43}{12} \right] - j \left[ \frac{-14}{3} \right] + k \left[ \frac{-7}{2} \right]$$

$$\int_0^1 (f \times v) = \frac{43}{12} i + \frac{14}{3} j - \frac{7}{2} k$$