5. $G(s) = \frac{16}{8s+4}$

Time constant $= 8/4 = 2$ (A/B)

Gain $= 16/4 = 4$ (T/B).

6. $\frac{\omega}{\theta}(s) = \frac{K_m}{T_m s + 2}$.

$K_m = 15s^{-1}$ and $T_m = 4s$.

$\therefore \frac{\omega}{\theta}(s) = \frac{15s^{-1}}{4s+2}$.

Gain $= 15/2 = K_m/B = 7.5s^{-1}$

Time constant $= 4/2 = T/B = 2$ seconds.

4. $\frac{\theta_o(s)}{\theta_i} = \frac{1}{Ts+1}$.

$\theta_i(t) = ct$

$\theta_i(s) = c/s^2$.

$$\theta_o(s) = \frac{\theta_i(s)}{(Ts+1)}$$

$$= \frac{c/s^2}{Ts+1}$$

$$= \frac{c}{s^2(Ts+1)}$$

$\theta_o(t) = c\{t - T(1 - e^{-t/T})\}$.

$\theta_e(t) = cT$.

$G(s) = 1/(3s+1)$ $c = 4$. $t = 2$.

After 2 seconds

$\theta_i = 4 \times 2 = 8mm$.

$\theta_o = 4(2 - 3(1 - e^{-2/3}))$

$= 4(2 - 3(0.486))$

$= 4(2 - 1.459)$

$= 4(0.540)$

$= \cancel{1.080}$ $2.161mm$.

$\cancel{\theta_e = CT}$

$\cancel{= 4 - 1.080}$

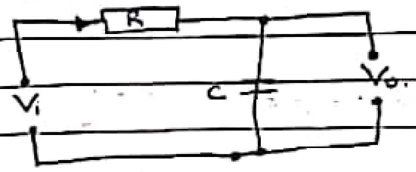
$\theta_e = 8 - 2.161$

$= 5.838\ mm$

Steady state error is CT

$= 4 \times 3$

$= 12 mm.$



$G(s) = \frac{V_o(s)}{V_i} = \frac{1}{Ts+1}$ where $T = RC$

$R = 47\Omega$ and $C = 20\mu F$ $V_i = 5\sin(2000t)$.

$V_i = IR + I/Cs.$

$V_i = I(R + 1/Cs)$

$V_o = I/Cs.$

$G(s) = \frac{V_o(s)}{V_i} = \frac{1/Cs}{I(R+1/Cs)} = \frac{1}{RCs+1}$

$T = RC.$

$G(s) = \frac{1}{Ts+1}$

$T = 47 \times 20 \times 10^{-6} = 940 \times 10^{-6} s.$

$\phi = -\tan^{-1}(\omega T) = -\tan^{-1}(2000 \times 940 \times 10^{-6}) = -62°$

$\frac{\theta_o}{\theta_i} = \frac{1}{\sqrt{1+T^2\omega^2}} = \frac{1}{\sqrt{1+(940\times10^{-6})^2 \cdot 2000^2}} = 0.47.$

Therefore

$\theta_o = (5 \times 0.47)\sin(2000t - 62°).$

$= 2.35\sin(2000t - 62°).$

2) $x_o/x_i = 1/(T^2s^2 + 2\delta Ts + 1)$

$T = 0.4$ seconds.

$\delta = 0.2$

$\theta_i = 6\sin(\omega t)$ at 2.5 rad/s.

Amplitude = ?

phaseshipt = ?

Constants C and D.

$$C = \frac{(1-\omega^2T^2)}{\{(1-\omega^2T^2)+(2\delta\omega T)^2\}}$$

$$C = \frac{(1-2.5^2 0.4^2)}{\{(1-2.5^2 0.4^2)+(2\times0.2\times2.5\times0.4)^2\}}.$$

$$C = \frac{0}{\{0+(0.4)^2\}}$$

$$C = 0$$

$$c = 0.$$

$$D = \frac{2\delta T\omega}{\{(1-T^2\omega^2)^2+(2\delta\omega T)^2\}}$$

$$= \frac{2\times0.2\times0.4\times2.5}{\{(1-0.4^2 2.5^2)^2+(2\times0.2\times2.5\times0.4)^2\}}$$

$$= \frac{0.4}{\{0+(0.4)^2}$$

$$= \frac{0.4}{0.16}$$

$$= 2.5.$$

$\phi = -\tan^{-1}(D/C) = -\tan^{-1}(\infty)$ therefore $\theta = 90^\circ$

$$|\theta_o/\theta_i| = \sqrt{D^2+C^2}$$

$$= \sqrt{2.5^2+0^2}$$

$$= 2.5.$$

$$\theta = 6\times2.5$$

$$= 15^\circ$$

1. $K_d = 0.03$ $F = 100N$

$K = 4KN/m$. $x = 16mm$.

$T = ?$

$$G(s) = \frac{x(s)}{F(s)} = \frac{1/K}{Ts+1}$$

$F = Kx + K_d\, dx/dt \longrightarrow i$

$$F = B\frac{dx}{dt} + Kx.$$

Laplace Transform of equ i

$F(s) = Kx + K_d s x$

$F(s) = x(K + K_d s)$

$$\frac{F(s)}{x} = K + K_d s$$

Taking inverse.

$$\frac{x}{F(s)} = \frac{1}{K + K_d s}$$

$$= \frac{1}{K_d s + K}.$$

$$\frac{x}{F(s)} = \frac{1/K}{(K_d/K)s + 1} = \frac{1/K}{Ts + 1}$$

$$T = K_d/K$$

$$= 0.03/4\times10^3$$

$T = 7.5\times10^{-6}$ seconds.

X_o after T seconds

$X_o = F/K\,(1 - e^{-1})$

$= 100/4\times10^3\,(1 - e^{-1})$

$= 100/4\times10^3 \times 0.632$

$= 0.025 \times 0.632$

$= 0.0158m$

$= 0.016m \simeq 16\times10^{-3} = 16mm.$

3.

2. $m = 0.5kg$

Specific ~~gravity~~ capacity $= 346$. $\theta_1 = 20°$

$\theta_2 = 120°c$

$\theta = 119°$

$$\frac{\theta_1}{\theta_2}(s) = \frac{1}{Ts+1}.$$

Show that $\theta = \theta_1 + (\theta_2 - \theta_1)(1 - e^{-t/T})$

$$\frac{\theta}{\theta_2} = \frac{1}{Ts+1}$$

$\theta = Ts\theta_2 + \theta_2$.

$\theta = \theta_2(Ts+1)$

$$\theta(s) = \frac{\theta_2}{Ts+1}$$

$$\theta(t) = \frac{1}{s(Ts+1)} \longrightarrow \text{(inverse laplace transform)}.$$

Rearranging

$$\theta(s) = \frac{1/T}{s(s+1/T)}$$

$\theta(t) = 1 - e^{-t/T}$

$\theta(t) = \theta_1 + (\theta_2 - \theta_1)(1 - e^{-t/T})$

$119 = 20 + (120-20)(1 - e^{-6/T})$

$99 = 100(1 - e^{-6/T})$

$0.99 = (1 - e^{-6/T})$

$0.99 - 1 = -e^{-6/T}$

$-0.01 = -e^{-6/T}$

$\ln 0.01 = -6/T$

$-4.605 = -6/T$

$T = 6/4.605$

$= 1.302$ mins. or 78.17 secs.

Thermal capacitance $C = mc = 0.5 \times 346 = 173$ J/K

$T = RC$.

$R = T/C$

$R = 78.17/173 = 0.451$ K/W.

5. $\frac{\omega}{K_m x} = \frac{1}{Ts+1}$

$T = \frac{J}{K_3}$ $\quad K_m = \frac{K_1 K_2}{K_3}$

$K_1, K_2, \omega, J, K_3, K_1 p$

Assuming no load torque

Torque $= K_1 p$

$= Ja + K_3 \omega$

$K_1 K_2 x(s) = Js\omega + K_3 \omega$

$(K_1 K_2 / K_3) x(s) = \omega \{ (J/K_3)s + 1 \}$.

$\frac{\omega}{K_m x} = \frac{1}{Ts+1}$

$\cancel{\frac{\omega}{K_m} = \frac{x}{Ts+1}}$

$K_1 K_2 x = J d\omega/dt + K_3 \omega$

$K_1 K_2 x(s) = \omega (Js + K_3)$

$K_m x(s) = \omega \{ Ts + 1 \}$.

$\frac{\omega}{K_m x} = \frac{1}{Ts+1}$

Substitute $x(s) = 1/s$.

$\frac{\omega}{K_m}(t) = \frac{1}{s(Ts+1)}$

Rearranging

$= \frac{1/T}{s(s+1/T)}$

$\frac{\omega}{K_m}(t) = (1 - e^{-t/T})$.

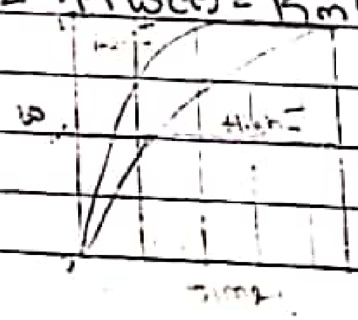
Sub $t = T$ $\omega(t) =$

$K_m(1 - e^{-1}) = 0.632 K_m$.

$= 0.632 \rightarrow 63.2\%$ change

Sub $t = 4T$ $\omega(t) = K_m(1 - e^{-4}) = 0.982 K_m$

$= 98.2\%$ change



ODJI Udochukwu Ebonez...
17/ENG04/064
Department: Electrical & Electronics Engineering
Linear Systems Assignment

Find the D.C gain and the time constant for the following transfer functions

i. $G(s) = \frac{2}{0.2s + 0.5}$

Take s to be 0

$= \frac{2}{0.2 \times 0 + 0.5}$

$= \frac{2}{0.5}$

D.C gain = 4

Time constant: $\frac{2/0.5}{(0.2/0.5)s + 1}$

$= \frac{4}{0.4s+1}$

Time constant = 4

ii. $G(s) = \frac{0.2}{0.05s + 0.1}$

At s = 0

D.C gain $= \frac{0.2}{0.1} = 2.$

Time constant = 0.05/0.1 = 0.5

iii. $G(s) = \frac{2}{3s+1}$

DC gain = 2

Time constant = 3.