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COURSE: MATH 104
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1. $\int \frac{3x-1}{(x-1)(x-2)(x-3)} dx$

Solution

$$\frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{A}{(x-1)} + \frac{B}{(x-2)} + \frac{C}{(x-3)}$$

$$\frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)}{(x-1)(x-2)(x-3)}$$

$$3x-1 \equiv A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)$$

when $x = 1$

$3(1) - 1 \equiv A(1-2)(1-3) + B(1-1)(1-3) + C(1-1)(1-2)$

$3 - 1 \equiv A(-1)(-2) + 0 + 0$

$2 = 2A$

$A = 1$

when $x = 2$

$3(2) - 1 \equiv A(2-2)(2-3) + B(2-1)(2-3) + C(2-1)(2-$

$6 - 1 = 0 + B(1)(-1) + 0$

$5 = -B$

$B = -5$

when $x = 3$

$3(3)-1 = A(3-2)(3-3) + B(3-1)(3-3) + C(3-1)(3-2)$
$9-1 = 0 + 0 + C(2\times1)$
$8 = 2C$
$C = \frac{4}{2}$

$\therefore \frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{1}{(x-1)} + \frac{(-5)}{(x-2)} + \frac{4}{(x-3)}$

$= \int \frac{1}{(x-1)} - \frac{5}{(x-2)} + \frac{4}{(x-3)} \, dx$

$= \ln(x-1) - 5\ln(x-2) + 4\ln(x-3) + C$

2. $\int \frac{x^2+x+1}{(x+2)(x^2+1)} \, dx$

$\frac{x^2+x+1}{(x+2)(x^2+1)} = \frac{A}{(x+2)} + \frac{Bx+C}{x^2+1}$

$\frac{x^2+x+1}{(x+2)(x^2+1)} = \frac{A(x^2+1) + (Bx+C)(x+2)}{(x+2)(x^2+1)}$

$x^2+x+1 \equiv A(x^2+1) + Bx^2 + 2Bx + Cx + 2C$
$x^2+x+1 = Ax^2 + A + Bx^2 + 2Bx + Cx + 2C$
$x^2+x+1 = Ax^2 + Bx^2 + 2Bx + Cx + 2C + A$

$x^2+x+1 = x^2(A+B) + x(2B+C) + (2C+A)$
$A+B = 1$ ..... equation(i)
$2B+C = 1$ ..... equation(ii)
$2C+A = 1$ ..... equation (iii)

$C = 1-2B$ ..... equation (iv)

$2(1-2B)+A=1$

$2-4B+A=1$

$4B-A=1$ ..... equation (v)

Solve simultaneously

$4B-A=1$

$+\ B+A=1$

$5B=2$

$\therefore B=\frac{2}{5}$

$A=1-\frac{2}{5}$

$A=\frac{3}{5}$

$C=1-\frac{4}{5}$

$C=\frac{1}{5}$

$\therefore \frac{3}{5}=A,\ \frac{2}{5}=B,\ \frac{1}{5}=C$

$$\therefore \int \frac{3}{5(x+2)} + \frac{2x+1}{5(x^2+1)}$$

$$= \frac{3}{5}\ln(x+2) + \frac{1}{5}\int \frac{2x+1}{x^2+1}$$

$$\int \frac{2x+1}{x^2+1} \quad \therefore \text{let } x=\tan\theta$$

$$\frac{dx}{d\theta} = \sec^2\theta$$

$$dx = \sec^2\theta\, d\theta$$

$$\int \frac{2x+1}{x^2+1}\,dx$$

$$= \int \frac{2(\tan\theta)+1}{\tan^2\theta+1} \cdot \sec^2\theta\, d\theta$$

$$= \int \frac{2\tan\theta + 1}{\sec^3\theta} \cdot \sec^2\theta \, d\theta$$

$$= \int 2\tan\theta + 1 \, d\theta$$

$$= 2\ln|\sec\theta| + \theta + c$$

$$= 2\ln|\sqrt{x^2+1}| + \tan^{-1}x + C$$

$$\frac{1}{5}(2\ln\sqrt{x^2+1} + \tan^{-1}x) + c$$

$$\therefore \int \frac{x^2+x+x}{(x+2)(x^2+1)} =$$

$$\frac{3}{5}\ln(x+2) + \frac{2}{5}\ln|\sqrt{x^2+1}| + \frac{1}{5}\tan^{-1}x +$$

3 $$\int \frac{x^2+1}{(x-3)(x-2)^2} \, dx$$

$$\frac{x^2+1}{(x-3)(x-2)^2} = \frac{A}{(x-3)} + \frac{B}{(x-2)} + \frac{C}{(x-2)^2}$$

$$\frac{x^2+1}{(x-3)(x-2)^2} = \frac{A(x-2)(x-2) + B(x-2)(x-3) + C(x-}{(x-3)(x-2)^2}$$

$$\therefore x^2+1 = A(x-2)^2 + B(x-2)(x-3) + C(x-3)$$

when $x = 3$

$$3^2-1 = A(3-2)(3-2) + B(3-2)(3-2 + C(3-3)$$

$$10 = A + 0 + 0$$

$$A = 10$$

N when $x = 2$

Ce $\therefore 2^2 + 1 = A(0)^2 + B(0)(-1) + C(2-3)$

De $\therefore 5 = -C$

M $C = -5$

$\therefore$ x has been 2 and 3, we can pick any any other number to be x to find B

$\therefore x = 1$

$1^2 + 1 = 10(1-2)^2 + B(1-2)(1-3) - 5(1-5)$

$2 = 10 + 2B + 10$

$2 - 10 - 10 = 2B$

$-18 = 2B$

$B = \frac{-9}{2}$

$\therefore$

$$\frac{x^2+1}{(x-3)(x-2)^2} = \frac{10}{(x-3)} + \frac{(-9)}{(x-2)} + \frac{(-5)}{(x-2)^2}$$

$$\therefore \int \frac{10}{(x-3)} - \frac{9}{(x-2)} - \frac{5}{(x-2)^2}\, dx$$

$$= 10\ln(x-3) - 9\ln(x-2) - 5\int \frac{1}{u^2}$$

$$= 10\ln(x-3) - 9\ln(x-2) + 5(x-2)^{-1} + C$$