MATHS 202 ASSIGNMENT

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Maths 202

1] $M = Pi - 6j - 8k$

$N = 4i - 3j - k$

$O = i - 3j + 2k$

a] M and N are perpendicular to each other

$M \cdot N = (pi - 6j - 3k) \cdot (4i + 3j - k)$

$= 4p - 18 + 3$

$= 4p - 15$

Since they are perpendicular

$4p - 15 = 0$

$\frac{\cancel{4}p}{\cancel{4}} = \frac{15}{4}$

$= \frac{15}{4}$

b] M, N and O are perpendicular

$$M \cdot (N \times O) = \begin{vmatrix} p & -6 & -3 \\ 4 & 3 & -1 \\ 1 & -3 & 2 \end{vmatrix}$$

$$= p \begin{vmatrix} 3 & -1 \\ -3 & 2 \end{vmatrix} + 6 \begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix} - 3 \begin{vmatrix} 4 & 3 \\ 1 & -3 \end{vmatrix}$$

$= p(6-3) + 6(8+1) - 3(-12-3)$

$= 3p + 6(9) - 3(-15) = 0$

$= 3p + 99 = 0$

$\frac{\cancel{3}p}{\cancel{3}} = \frac{-99}{3}$

$p = \frac{-\cancel{99}\,33}{\cancel{3}\,1}$

$p = -33$

2] $\bar{v} = (3i + 2j + 5k) + (2i - j + 6k) + (5i + 2j \ldots$

$v = 10i + 3 + 8k$

$a_x = 10,\ a_y = 3,\ \text{and } a_z = 8$

$|v| = \sqrt{10^2 + 3^2 + 8^2}$

$= \sqrt{100 + 9 + 64}$

$= \sqrt{173} = 13.15$

i] The direction cosines are

$\cos\alpha = \frac{a_x}{|v|} = \frac{10}{13.15} = 0.228$

$\cos\gamma = \frac{a_z}{|v|} = \frac{8}{13.15} = 0.608$

ii] Unit vector

$u_v = \frac{v}{|v|} = \frac{10i + 3j + 7k}{13.15}$

3] $F = 3ui + u^2j + (u+2)k$

$V = 2ui - 3uj + (u-2)k$

$$(F \times V) = \begin{vmatrix} I & J & K \\ 3u & u^2 & (u+2) \\ 2 & -3u & (u-2) \end{vmatrix}$$

$$= i\begin{vmatrix} u^2 & (u+2) \\ -3u & (u-2) \end{vmatrix} - j\begin{vmatrix} 3u & (u+2) \\ 2u & (u-2) \end{vmatrix} + k\begin{vmatrix} 3u & u^2 \\ 2u & -3u \end{vmatrix}$$

$= i(u^2(u-2) - (-3u(u+2)) - j(3u(u-2) - 2u(u+2)) + k(-3$
$-4u$

$= i[u^3 - 2u^2 - (-3u - 6u)] - j(3u^2 - 6u - 3u^2 - 4u) + 6$
$(-2u^3 - 3u^2)$

$= i(u^3 - 2u^2 + 3u^2 + 6u) - j(u^2 - 100) + k(-2u^3 - 9u^3)$

$= i(u^3 + u^2 + 6u) - j(u^2 - 10u) + k(-2u^3 - 9u^2)$

$$\int_0^1 (f \times v) = \int i\,(u^3+u^2+6u) - \int j\,(u^2-10u) + \int k\,(-2u^3-9u^2)$$

$$= i\int u^3+u^2+6u - 5\int u^2 - 10u + k\int -2u^3 - 9u^2$$

$$= \left[\frac{u^4}{4} + \frac{u^3}{3} + \frac{\cancel{6}^{3}u^2}{\cancel{2}_1}\right] - j\left[\frac{u^3}{3} - \frac{\cancel{10}^{5}u^2}{\cancel{2}_1}\right] + k\left[\frac{-\cancel{2}^{1}u^4}{\cancel{4}_2} - \frac{\cancel{9}^{3}u^3}{\cancel{3}_1}\right] + c$$

$$= \left[\frac{u^4}{4} + \frac{u^3}{3} + 3u^2\right] - j\left[\frac{u^3}{3} - 5u^2\right] + k\left[\frac{-u^4}{2} - 3u^3\right] + c$$

$$\int_0^1 (f \times v) = i\left[\frac{(1)^4}{4} + \frac{(1)^3}{3} + 3(1)^2\right] - j\left[\frac{(1)^3}{3} - 5(1)^2\right] + k$$

$$\left[\frac{-(1)^4}{2} - 3(1)^3\right] + c - (0 + c)$$

$$\int_0^1 (f \times v) = i\left[\frac{1}{4} + \frac{1}{3} + 3\right] - j\left(\frac{1}{3} - 5\right) + k\left(\frac{-1}{2} - 3\right)$$

$$+c-c$$

$$\int_0^1 (f \times v) = i\left(\frac{43}{12}\right) - j\left(\frac{-14}{3}\right) + k\left(\frac{-7}{2}\right)$$

$$\int_0^1 (f \times v) = \frac{43}{12}\,i + \frac{14}{3}\,j - \frac{7}{2}\,k$$