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Mechatronics Engineering

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(1) $\int \frac{(3x-1)}{(x-1)(x-2)(x-3)}\,dx$

$$\frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$$

$3x-1 = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)$

When $x = 2$

$3(2)-1 = B(2-1)(2-3)$

$6-1 = B(1)(-1)$

$5 = -B$

$B = -5$

when $x = 3$

$3(3)-1 = C(2)(1)$

$9-1 = 2C$

$8 = 2C$

$C = 4$

when $x = 1$

$3(1)-1 = A(-1)(-2)$

$2 = 2A$

$A = 1$

$$\int \frac{3x-1}{(x-1)(x-2)(x-3)}\,dx = \int \frac{1}{x-1}\,dx - \int \frac{5}{x-2}\,dx + \int \frac{4}{x-3}$$

$$= \ln x - 1 - 5\ln(x-2) + 4\ln(x-3) + C$$

$Bx + 2Bx + Cx + 2C$

(2) $\int \frac{x^2+x+1}{(x+2)(x^2+1)}\,dx$

$$\frac{x^2+x+1}{(x+2)(x^2+1)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+1}$$

$x^2+x+1 = A(x^2+1) + (Bx+C)(x+2)$

let $x = -2$

$4-2+1 = A(4+1)$

$3 = 5A$

$A = 3/5$

Expansion of the R.H.S

$Ax^2 + A + Bx^2 + 2Bx + Cx + 2C$

$Ax^2 + Bx^2 + 2Bx + Cx + A + 2C$

~~Compare R.H.S to L.H.S~~

$x^2(A+B) + (2B+C)x + A + 2C$

Compare R.H.S to L.H.S

$A+B = 1$ ——— (i)

$2B+C = 1$ ——— (ii)

$A+2C = 1$ ——— (iii)

Subst A into eqn (i)

$3/5 + B = 1$

$B = 1 - 3/5 = 2/5$

Subst B into eqn (iii)

~~$A + 2 \quad 2/5 + 2C$~~

$$2C = \frac{2}{5}$$

$$C = \frac{2}{5} \times \frac{1}{2}$$

$$2\left(\frac{2}{5}\right) + C = 1$$

$$\frac{4}{5} + C = 1$$

$$C = 1 - \frac{4}{5}$$

$$C = \frac{1}{5}$$

$$\int \frac{x^2+x+1}{(x+2)(x^2+1)}\,dx \Longrightarrow \frac{3}{5}\int \frac{dx}{x+2} + \frac{1}{5}\int \frac{2x+1}{x^2+1}$$

$$\Rightarrow \frac{3}{5}\ln(x+2) + \frac{1}{5}(\tan^{-1}x^2)$$

$$\frac{3}{5}\ln(x+2) + \frac{2x}{5}\ln(x^2+1) + \frac{1}{5}(\tan^{-1}x) + C$$

$$\frac{3}{5}\int \frac{1x}{x+2} + \frac{1}{5}\left(\int \frac{2x}{x^2+1}\,dx + \int \frac{1}{x^2+1}\,dx\right)$$

$$\frac{3}{5}\ln(x+2) + \frac{1}{5}(\ln x^2+1) + \frac{1}{5}(\tan^{-1}x) + C$$

(3) $\int \frac{(x^2+1)}{(x-3)(x-2)^2}\,dx$

$$\frac{x^2+1}{(x-3)(x-2)^2} = \frac{A}{x-3} + \frac{B}{(x-2)} + \frac{C}{(x-2)^2}$$

$$\frac{x^2+1}{(x-3)(x-2)^2} = \frac{A(x+2)^2 + B(x-3)(x-2) + C(x-3)}{(x-3)(x-2)^2}$$

$x^2+1 = A(x+2)^2 + B(x-3)(x-2) + C(x-3)$

~~$x^2+1 = A(x^2+4x+4) + B$~~

$x^2+1 = A(x^2+4x+4) + B(x^2-5x+6) + C(x-3$

$x^2+1 = Ax^2+4Ax+4A+Bx^2-5Bx+6B+Cx-3C$

$x^2+1 = Ax^2+Bx^2+4Ax-5Bx+4A+6B-3C$

Compare firstly let $x=3$

$9+1 = 25A$

$A = \frac{10}{25}$

$x^2+1 = (A+B)x^2 + x(4A-5B+C) + 4A+6B-3C$

Compare

$A+B=1$ ---- (i)

$4A-5B+C=0$ --- (ii)

$4A+6B-3C=1$ --- (iii)

eliminate eqn (ii) and (iii)

$4A-5B+C=0$

$4A+6B-3C=1$

$B-2C=1$

subst $A = 1-B$

subst A into eqn (i)

$\frac{10}{25} + B = 1$

$B = 1 - \frac{2}{5}$

$B = \frac{3}{5}$

Subst A and B into eqn (ii)

$4 \times \frac{2}{5} - 5 \times \frac{3}{5} + C = 0$

$\frac{8}{5} - \frac{15}{5} + C = 0$

$-\frac{7}{5} + C = 0$

$C = \frac{7}{5}$

$(x+2)(x-1)$

$x^2 - 2x + 3x - 6$

$x^2 + x - 6$

$Bx^2 - 3Bx + 6B$

$\int \frac{x^2+1}{(x-3)(x-2)^2} dx = \frac{2}{5}\int \frac{1\,dx}{x-3} + \frac{3}{5}\int \frac{dx}{x-2} + \frac{7}{5}\int \frac{dx}{(x-2)^2}$

$= \frac{2}{5}\ln(x-3) + \frac{3}{5}\ln(x-2) + \frac{7}{5}(x+2)^{-1} + C$

(4) $\int \frac{x^3+x^2+x+1}{x-1}$

$x^2 + 2x + 3$

$x-1 \;\overline{)\; x^3 + x^2 + x + 1}$

$-(x^3 - x^2)$

$2x^2 + x + 1$

$-(2x^2 - 2x)$

$3x + 1$

$-(3x - 3)$

4

$Bx(x+2) + C(x+2)$

$Bx^2 + 2Bx + Cx + 2C$

$$\therefore \int x^2\,dx + \int 2x\,dx + \int 3\,dx + \int \frac{4}{x-1}\,dx$$

$$\frac{x^3}{3} + x^2 + 3x + 4\ln(x-1) + C$$