MAT 104
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1 $\int \frac{3x-1}{(x-1)(x-2)(x-3)}\,dx$

$$\frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{A}{(x-1)} + \frac{B}{(x-2)} + \frac{C}{(x-3)}$$

$3x-1 = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)$

when $x-1=0 \quad x=1$

$3(1)-1 = A(1-2)(1-3) + B(0)(x-3) + C(x=0)(x-2)$

$2 = 2A$

$A = 2/2$

$A = 1$

when $x-2=0 \quad x=2$

$3(2)-1 = A(0)(x-3) + B(2-1)(2-3) + C(x-1)(0)$

$5 = 0 + -B + 0$

$5 = -B$

$B = -5$

When $x-3=0 \quad x=3$

$3(3)-1 = A(x-2)(0) + B(x-1)(0) + C(3-1)(3-2)$

$8 = 0 + 0 + 2C$

$8 = 2c$

$C = 4$

$$\int \frac{3x-1\,dx}{(x-1)(x-2)(x-3} = \int \frac{1}{x-1} + \frac{-5}{x-2} + \frac{4}{x-3}\,dx$$

$$= \ln(x-1) - 5\ln(x-2) + 4\ln(x-3) + C$$

2 $\int \frac{x^2+x+1}{(x+2)(x^2+1)}\,dx$

$$\frac{x^2+x+1}{(x+2)(x^2+1)}dx = \frac{A}{(x+2)} + \frac{Bx+C}{(x^2+1)}$$

$$x^2+x+1 = A(x^2+1) + (Bx+C)(x+2)$$

· When ~~$x^2+1=0$~~

$$x^2+x+1 = Ax^2+A+Bx^2+2Bx+Cx+2C$$

$$x^2 = Ax^2+Bx^2$$

$$1 = A+B \quad ----①$$

$$x = Cx+2Bx$$

$$1 = C+2B \quad ----②$$

~~$C = 1$ ----④~~

~~$1 = A+2B+$~~

$$1 = A+2C \quad ----③$$

$1 = A+B$

$1 = C+2B$

$1 = A+2C$

$1 = 2B+C \times 2$

$1 = A+2C \times 1$

$2 = 4B+2C$

$-1 = A+2C$

$1 = 4B-A \quad ----④$

$1 = 4B-A$

$+1 = A+B$

$2 = 5B+0$

$B = 2/5$

$1 = A+B$

$1 = A+\frac{2}{5}$

$A = 1-2/5 = 3/5$

$A = \frac{3}{5}$

$1 = A+2C$

$1 = \frac{3}{5}+2C$

$1-3/5 = 2C$

$2/5 = 2C$

$$C = \frac{4}{5}$$

$$\int \frac{x^2+x+1}{(x+2)(x^2+1)}\,dx = \int \frac{3}{5(x+2)} + \frac{2x}{5(x^2+1)} + \frac{4}{5(x^2+1)}\,dx$$

$$\int \frac{3}{5(x+2)} + \frac{2x+4}{5(x^2+1)}\,dx$$

$$= \frac{3}{5}\ln(x+2) + \frac{1}{5}\ln(x^2+1) + \frac{4}{5}\tan^{-1}x + C$$

3 $\int \frac{x^2+1}{(x-3)(x-2)^2}\,dx =$

$$\frac{x^2+1}{(x-3)(x-2)^2} = \frac{A}{x-3} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$$

$$x^2+1 = A(x-2)^2 + B(x-3)(x-2) + C(x-3)$$

$$x^2+1 = A(x^2-4x+4) + B(x^2-5x+6) + C(x-3)$$

$$x^2+1 = Ax^2 - 4Ax + 4A + Bx^2 - 5Bx + 6B + Cx - 3C$$

$x^2 + 1 = x^2 = Ax^2 + Bx^2$ $\quad 1 = A + B$ ---①

$0x = -4Ax - 5Bx + Cx$ $\quad 0 = -4A - 5B + C$ ---②

$1 = 4A + 6B - 3C$ ---③

$0 = -4A - 5B + C$

$-C = -4A - 5B$ $\quad C = 4A + 5B$ ---④

$1 = 4A + 6B - 3C$

$1 = 4A + 3B - 3(4A + 5B)$

$1 = 4A + 3B - 12A - 15B$

$1 = -8A - 12B$ ---⑤

$1 = A + B \quad \times -8$

$1 = -8A - 12B \quad \times 1$

$-8 = -8A - 8B$

$-1 = -8A - 12B$

$-9 = 0 + 4B$

$-9 = 4B$

$B = -9/4$

$1 = A + B$

$1 = A - \frac{9}{4}$ $\quad A = 1 + 9/4$

$A = 13/4$

$C = 4A + 5B$

$C = 4 \cdot \frac{13}{4} + 5 \cdot \frac{-9}{4}$

$C = 13 - \frac{45}{4}$

$C = 7/4$

$$\int \frac{x^2+1}{(x-3)(x-2)^2} = \int \frac{13}{4(x-3)} + \frac{-9}{4(x-2)} + \frac{7}{4(x-2)^2} \, dx$$

$$= \frac{13}{4} \ln(x-3) - \frac{9}{4} \ln(x-2) + \int \frac{7}{4(x-2)^2} \, dx$$

$\int \frac{7}{4} \int \frac{1}{u^2} \, dx \quad u = x-2$

$du/dx = 1 \quad dx = 1\,du$

$\frac{7}{4} \int u^{-2} \cdot du$

$\frac{7}{4} \frac{u^{-2+1}}{-2+1} + C \quad \frac{7}{4} \frac{u^{-1}}{-1} + C$

$-\frac{7}{4} u^{-1} + C \quad \frac{7}{4} (x-2)^{-1} + C$

$= \frac{13}{4} \ln(x-3) - \frac{9}{4} \ln(x-2) + \frac{7}{4} (x-2)^{-1} + C$

4 $\int \frac{x^3 + x^2 + x + 1}{x-1} \, dx$

$$
\begin{array}{r|l}
x-1 & x^3+x^2+x+1 \\
 & -x^3-x^2 \\
\hline
 & 2x^2+x+1 \\
 & -2x^2-2x \\
\hline
 & 3x+1 \\
 & -3x-3 \\
\hline
 & 4
\end{array}
\qquad x^2+2x+3
$$

$$\frac{x^3+x^2+x+1}{x-1} = x^2+2x+3 + \frac{4}{x-1}$$

$$\int x^2+2x+3 + \frac{4}{x-1}\, dx$$

$$\frac{x^3}{3} + \frac{2x^2}{2} + 3x + 4\ln(x-1) + C$$

$$\frac{x^3}{3} + x^2 + 3x + 4\ln(x-1) + C$$