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Find the integral of the following

- 1)  $\frac{3x-1}{(x-1)(x-2)(x-3)} dx$
- 2)  $\frac{(x^2+x+1)}{(x+2)(x^2+1)} dx$
- 3)  $\frac{(x^2+1)}{(x-3)(x-2)^2} dx$
- 4)  $\frac{x^3+x^2+x+1}{(x-1)} dx$

Solution

$$\frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$$
$$\frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)}{(x-1)(x-2)(x-3)}$$

$$3x-1 = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)$$

$$3x-1 = Ax^2 - 5Ax + 6A + Bx^2 - 4Bx + 3B + Cx^2 - 3Cx + 2C$$

$$3x-1 = (A+B+C)x^2 + (-5A-4B-3C)x + (6A+3B+2C)$$

Comparing / Equating Coefficients

$$A+B+C=0 \quad \text{--- (i)}$$

$$-5A-4B-3C=3 \quad \text{--- (ii)}$$

$$6A+3B+2C=-1 \quad \text{--- (iii)}$$

$$A+B+C=0$$

$$A = -B - C \quad \text{--- (iv)}$$

put equation iv in (ii) & (iii)

$$-5A - 4B - 3C = 3 \implies -5(-B-C) - 4B - 3C = 3$$

$$6A + 3B + 2C = -1 \implies 6(-B-C) + 3B + 2C = -1$$

$$B + 2C = 3 \quad \text{--- (v)} \quad \times -4$$

$$-3B - 4C = -1 \quad \text{--- (vi)} \quad \times 2$$

$$-6B - 8C = -12$$

$$2B - 8C = -2$$

$$B = -5$$

put  $B = -5$  in equation (v)



$$B + 2C = 3$$

$$-5 + 2C = 3$$

$$2C = 5 + 3$$

$$C = 4$$

Put  $B = -5$  &  $C = 4$  in equation (1)

$$A + B + C = 0$$

$$A + 5 + 4 = 0$$

$$A - 1 = 0$$

$$A = 1$$

$$\int \frac{(3x-1)}{(x-1)(x-2)(x-3)} dx = \int \frac{1}{(x-1)} dx + \int \frac{-5}{(x-2)} dx + \int \frac{4}{(x-3)} dx$$
$$= \ln(x-1) - 5 \ln(x-2) + 4 \ln(x-3) + C$$

$$2) \frac{x^2 + x + 1}{(x+2)(x^2+1)} dx = \frac{A}{(x+2)} + \frac{(Bx+C)}{(x^2+1)}$$

$$\frac{x^2 + x + 1}{(x+2)(x^2+1)} dx = \frac{A(x^2+1)}{(x+2)(x^2+1)} + \frac{(Bx+C)(x+2)}{(x^2+1)}$$

$$x^2 + x + 1 = Ax^2 + A + Bx^2 + 2Bx + Cx + 2C$$

$$x^2 + x + 1 = (A+B)x^2 + (2B+C)x + (A+2C)$$

Equating Coefficients

$$A + B = 1 \quad \text{--- (i)}$$

$$2B + C = 1 \quad \text{--- (ii)}$$

$$A + 2C = 1 \quad \text{--- (iii)}$$

$$A = 1 - B \quad \text{--- (iv)}$$

put equation (iv) into (ii)

$$A + 2C = 1$$

$$1 - B + 2C = 1$$

$$-B + 2C = 0$$

$$B = 2C$$

put  $B = 2C$  in equation (ii)

$$2B + C = 1$$

$$2(2C) + C = 1$$

$$4C + C = 1$$

$$\frac{5C}{5} = \frac{1}{5}$$

$$\therefore C = \frac{1}{5}$$

put  $C = \frac{1}{5}$  in equation (ii)

$$\begin{aligned} A + 2C &= 1 \\ A + 2\left(\frac{1}{5}\right) &= 1 \\ A + \frac{2}{5} &= 1 \\ A &= \frac{3}{5} \end{aligned}$$

put  $A = \frac{3}{5}$  in equation (1)

$$\begin{aligned} A + B &= 1 \\ \frac{3}{5} + B &= 1 \\ B &= 1 - \frac{3}{5} \\ B &= \frac{2}{5} \end{aligned}$$

$$\therefore \int \frac{(x^2 + x + 1)}{(x+2)(x^2+1)} dx = \frac{3}{5} \ln|x+2| + \frac{2x+1}{5} \ln|x^2+1| + C$$

$$3. \frac{(x^2+1)}{(x-3)(x-2)^2} dx = \frac{A}{(x-3)} + \frac{B}{(x-2)} + \frac{C}{(x-2)^2}$$

$$\frac{x^2+1}{(x-3)(x-2)^2} = \frac{A(x-2)^2 + B(x-3)(x-2) + C(x-3)}{(x-3)(x-2)^2}$$

$$x^2+1 = Ax^2 - 4Ax + 4A + Bx^2 - 5Bx + 6B + Cx - 3C$$

$$x^2+1 = (A+B)x^2 + (-4A-5B+C)x + (4A+6B-3C)$$

Equating the Coefficients

$$A + B = 1 \quad \text{--- (i)}$$

$$-4A - 5B + C = 0 \quad \text{--- (ii)}$$

$$4A + 6B - 3C = 1 \quad \text{--- (iii)}$$

$$A = 1 - B \quad \text{--- (iv)}$$

put equation (iv) into (ii) & (iii)

$$-4A - 5B + C = 0 \iff -4(1-B) - 5B + C = 0 \implies -B + C = 4 \quad \text{--- (v)}$$

$$4A + 6B - 3C = 1 \implies 4(1-B) + 6B - 3C = 1 \implies 2B - 3C = -3 \quad \text{--- (vi)}$$

$$\begin{array}{r} 2 \cdot 3B - 3C = -12 \\ -2B - 3C = -3 \\ \hline \end{array}$$

$$4B - 6C = -9$$

$$B = -9$$

put B in equation (i)

$$A + B = 1$$

$$A + (-9) = 1$$

$$A = 10$$

put B in equation (v)

$$-B + C = 4$$

$$-(-9) + C = 4$$

$$C = 4 - 9$$

$$C = -5$$

$$\begin{aligned} \therefore \int \frac{x^2+1}{(x-3)(x-2)} dx &= \int \frac{10}{x-3} dx + \int \frac{-9}{x-2} dx + \int \frac{-5}{(x-2)^2} dx \\ &= 10 \ln(x-3) - 9 \ln(x-2) + 5(x-2)^{-1} + C \\ &= 10 \ln(x-3) - 9 \ln(x-2) + \frac{5}{x-2} + C \end{aligned}$$

$$4 \int \frac{x^3+x^2+x+1}{x-1} dx$$

$$\begin{array}{r} x^2 + 2x + 3 \\ x-1 \overline{) x^3 + x^2 + x + 1} \\ \underline{x^3 - x^2} \phantom{+ 1} \\ 2x^2 + x \phantom{+ 1} \\ \underline{2x^2 - 2x} \phantom{+ 1} \\ 3x + 1 \\ \underline{3x - 3} \\ 4 \end{array}$$

$$\therefore \int (x^2 + 2x + 3) dx + \int \frac{4}{x-1} dx$$

$$\Rightarrow \frac{x^3}{3} + \frac{2x^2}{2} + 3x + 4 \ln(x-1) + C$$

$$= \frac{x^3}{3} + x^2 + 3x + 4 \ln(x-1) + C$$

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