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Mechatronics Engineering

1 $\frac{3x-1}{(x-1)(x-2)(x-3)}dx = \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x-5}$

$= A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)$

when $x=3$

$3(3)-1 = C(3-1)(3-2)$

$8 = 2C \quad \therefore C = 4$

when $x=2$

$3(2)-1 = B(2-1)(2-5)$

$5 = B \times -1 \quad \therefore B = -5$

when $x=1$

$3-1 = A(1-2)(1-3)$

$2 = A \times 2 \quad \therefore A = 1$

$\therefore \int \frac{3x-1}{(x-1)(x-2)(x-3)}dx = \frac{1}{(x-1)} + \frac{-5}{(x-2)} + \frac{4}{(x-3)}$

$= \ln(x-1) - 5\ln(x-5) + 4\ln(x$

$= \ln(x-1) - 5\ln(x-5) + 4\ln(x-3) + C$

2 $\int \frac{x^2+x+1}{(x+2)(x^2+1)}dx = \frac{A}{x+2} + \frac{Bx+C}{x^2+1}$

$\therefore x^2 + x + 1 = A(x^2+1) + (Bx+C)(x+2)$

when $x=-2$

$(-2)^2 - 2 + 1 = A(4+1)$

$3 = 5A \quad \therefore A = 3/5$

$x^2 + x + 1 = Ax^2 + A + Bx^2 + 2Bx + Cx + 2C$

$\therefore x^2(A+B) + (2B+C)x + A + 2C$

$\therefore A+B=1$, $2B+C=1$ & $A+2C=1$

But $A = 3/5$

$\therefore B = 1 - 3/5 = 2/5$

$2 \times \frac{2}{5} + C = 1$

$C = 1 - 4/5 = \frac{1}{5}$

$\therefore \int \frac{x^2+x+1}{(x+2)(x^2+1)} = \int \frac{3}{5x+10} + \int \frac{2x+1}{5(x^2+1)}$

$I = \frac{3}{5}\ln(x+2) + \frac{1}{5}\ln(x^2+1) + \frac{1}{5}\tan^{-1}(x) + C$

3 $\frac{x^2+1}{(x-3)(x-2)^2} = \frac{A}{x-3} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$

$\therefore x^2 + 1 = A(x-2)^2 + B(x-3)(x-2) + C(x-3)$

when $x=3$

$3^2 + 1 = A(3-2)^2$

$A = 10$

when $x = 2$

$5 = -1C$

$\therefore C = -5$

$x^2 + 1 = Ax^2 - 4Ax + 4A + Bx^2 - 5Bx + 6B + Cx - 3C$

$\therefore 1 = A + B \quad 1 = 4A + 6B - 3$

$1 = 10 + B$

$\therefore B = -9$

$1 = 4(10) + 6(-9) - 3C$

$1 = 40 - 54 - 3C$

$3C = -1 + 60 - 54$

$= -15$

$\therefore C = -5$

$\therefore \int \frac{x^2+1}{(x-3)(x-2)^2} = \int \frac{10}{(x-3)}dx + \int \frac{-9}{(x-3)}dx + \int \frac{-5}{(x-2)^2}$

$= 10\ln(x-3) - 9\ln(x-2) + 5(x-2)^{-1} + C$

4) $\int x^3 + x^2 + x + 1 \, dx$

$$\int \frac{x^3+x^2+x+1}{x-1}\,dx$$

$$x-1 \;\overline{)\; x^3+x^2+x+1}$$

$$x^2+2x+3$$

$$-x^3-x^2$$

$$2x^2+x$$

$$-2x^2-2x$$

$$3x+1$$

$$3x-4$$

$$4$$

$$\therefore \int \frac{x^3+x^2+x+1}{x-1} = \int (x^2+2x+3)\,dx + \int \frac{4}{x-1}$$

$$= \frac{x^3}{3} + x^2 + 3x + 4\ln(x-1) + C$$