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1) $\frac{3x-1}{(x-1)(x-2)(x-3)}\,dx$

Solution:

$$\frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{A}{(x-1)} + \frac{B}{x-2} + \frac{C}{(x-3)}$$

multiply through by $(x-1)(x-2)(x-3)$

$3x-1 = (x-2)(x-3)A + (x-1)(x-3)B + (x-1)(x-2)C$

$3x-1 = Ax^2 - 5Ax + 6A + Bx^2 - 4Bx + 3B + Cx^2 - 3Cx + 2C$

$3x-1 = Ax^2 + Bx^2 + Cx^2 - 5Ax - 4Bx - 3Cx + 6A + 3B + 2C$

$3x-1 = (A+B+C)x^2 + (5A-4B-3C)x + (6A+3B+2C)$

$-1 = 6A + 3B + 2C \; -(1)$

$3 = -5A - 4B - 3C \; -(2)$

$0 = A + B + C \; — (3)$

Sub eqn (3) into (2) $A = -B - C \; -(4)$

Sub eqn (4) into eqn (1) & (2)

$6(-B-C) + 3B + 2C = -1$

$-5(-B-C) - 4B - 3C = 3$

$-3B - 4C = -1 \; --- (5)$

$B + 2C = 3 \; --- (6)$

$\cancel{B = 3-2x}$ $B = 3 - 2C$

Subst B into eqn 5

$-3(3-2C) - 4C = -1$

$-9 + 6C - 4C = -1$

$-9 + 2c = -1$

$C = 4$

sub C in eqn (6)

$B + 2(4) = 3$

$B = 3 - 8$

$B = -5$

sub B and c into eqn 6)

$6A + 3B + 2C = -1$

$6A + 3(-5) + 2(4) = 1$

$\cancel{6A}\ 6A - 15 + 8 = -1$

$6A - 7 = -1$

$6A = 7 + 1$

$\cancel{A=}\ A = -1$

$(A, B, C) = 1, -5, -1$

$$\frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{1}{x-1} - \frac{5}{(x-2)} - \frac{1}{(x-3)}$$

$$\int \frac{3x-1}{(x-1)(x-2)(x-3)}\,dx = \int \frac{1}{x-1}\,dx - \int \frac{5}{x-x}\,dx - \int \frac{1}{x-3}$$

$= \ln(|x-1|) - 5\ln(|x-2|) - \ln(|x-3|) + c$

2) $\int \frac{(x^2+x+1)}{(x+2)(x^2+1)}\,dx$

Solution

$$\frac{x^2+x+1}{(x+2)(x^2+1)} = \frac{A}{x+2} + \frac{B+C}{(x^2+1)}$$

$x^2+x+1 = (x^2+1)A + (x+2)(Bx+C)$

$x^2+x+1 = Ax^2 + A + Bx^2 + Cx + 2Bx + 2C$

$x^2+x+1 = Ax^2 + Bx^2 + Cx + 2Bx + 2C + A$

$x^2+x+1 = (A+B)x^2 + (C+2B)x + (2C+A)$

$1 = A + 2C$

$1 = C + 2B$

$1 = A + B$

$(A, C, B) = (3/5), (2/5), (1/5)$

$$\frac{x^2+x+1}{(x+2)(x^2+1)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+1}$$

$$= \frac{3}{5(x+2)} + \frac{2x+1}{5(x^2+1)}$$

$$= \int \frac{3}{5(x+2)}\,dx + \int \frac{2x+1}{5x^2+1}\,dx$$

$\frac{3}{5} x \ln(|x+2|) + \frac{1}{5} x \ln(|x^2+1|)$

3. $\frac{x^2+1}{(x-3)(x-2)^2} dx$

solution

$$\frac{x^2+1}{(x-3)(x-2)^2} = \frac{A}{(x-3)} + \frac{B}{(x-2)} + \frac{C}{(x-2)^2}$$

$x^2+1 = (x-2)^2 A + (x-3)(x-2)B + (x-3)C$

$x^2+1 = Ax^2 - 4Ax + 4A + Bx^2 - 5Bx + 6B + Cx - 3C$

$x^2+1 = Ax^2 + Bx^2 - 4Ax - 5Bx + Cx + 4A + 6B - 3C$

$x^2+1 = (A+B)x^2 + (-4A - 5B + C)x + (4A + 6B - 3C)$

$1 = 4A + 6B - 3C$

$0 = -4A - 5B + C$

$1 = A + B$

$(A, B, C) = (10, -9, -5)$

$$\frac{x^2+1}{(x-3)(x-2)^2} = \frac{10}{(x-3)} - \frac{9}{(x-2)} - \frac{5}{(x-2)^2}$$

$\therefore \int 10/(x-3)\,dx - \int 9/(x-2)\,dx - \int 5/(x-2)^2\,dx$

$= 10\ln(|x-3|) - 9\ln(|x-2|) + 5/x-2$

$= 10\ln(|x-3|) - 9\ln(|x-2|) + 5/x-2\ dx.$

4) $\frac{(x^3+x^2+x+1)}{x-1} dx$

Solution

$$\frac{x^3}{x-1} + \frac{x^2}{x-1} + \frac{x}{x-1} + \frac{1}{x-1}\ dx.$$

$$\int \frac{x^3}{x-1}\,dx + \int \frac{x^2}{x-1}\,dx + \int \frac{x}{x-1}\,dx + \int \frac{1}{x-1}\,dx$$

$$\frac{2x^3+3x^2+6x-11}{6} + \ln(|x-1|) + \frac{x^2}{2} + x + \ln(x-1)$$

$$+ (x-1 + \ln 6(|x-1|) + \ln(|x+1|) + c$$

$$= \frac{2x^3+3x^2+6x+11}{6} + 4\ln(|x-1|) + c$$