Oyigbenu Meshack
18/ENG02/084.

Define 3 matrices A, B and C

$$A = \begin{pmatrix} 1 & -3 & 6 \\ 4 & 0 & 2 \\ 8 & 5 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 1 & 2 \\ 1 & -2 & 1 \\ 0 & 3 & -4 \end{pmatrix} \quad C = \begin{pmatrix} 0 & 4 & 3 \\ 6 & -7 & 1 \\ 1 & -3 & 2 \end{pmatrix}$$

1 Find Linear transformation of A if vector $x = (a, b, c)$

$$T(x) = Ax$$

$$T(x) = \begin{pmatrix} 1 & -3 & 6 \\ 4 & 0 & 2 \\ 8 & 5 & 1 \end{pmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$T(x) = \begin{bmatrix} 1 & -3 & 6 \\ 4 & 0 & 2 \\ 8 & 5 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$= a\begin{bmatrix} 1 \\ 4 \\ 8 \end{bmatrix} + b\begin{bmatrix} -3 \\ 0 \\ 5 \end{bmatrix} + c\begin{bmatrix} 6 \\ 2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} a \\ 4a \\ 8a \end{bmatrix} + \begin{bmatrix} -3b \\ 0 \\ 5b \end{bmatrix} + \begin{bmatrix} 6c \\ 2c \\ c \end{bmatrix}$$

$$= \begin{bmatrix} a + (-3b) + 6c \\ 4a + 0 + 2c \\ 8a + 5b + c \end{bmatrix}$$

$$T(x) = \begin{bmatrix} a - 3b + 6c \\ 4a + 2c \\ 8a + 5b + c \end{bmatrix}$$

Hence transformation of $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$ gives $\begin{bmatrix} a - 3b + 6c \\ 4a + 2c \\ 8a + 5b + c \end{bmatrix}$

$$B+C = \begin{pmatrix} 1 & 1 & 2 \\ 1 & -2 & 1 \\ 0 & 3 & -4 \end{pmatrix} + \begin{pmatrix} 0 & 4 & 3 \\ 6 & -7 & 1 \\ 1 & -3 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 5 & 5 \\ 7 & -9 & 2 \\ 1 & 0 & -2 \end{pmatrix}$$

$$(B+C)^T = \begin{pmatrix} 1 & 7 & 1 \\ 5 & -9 & 0 \\ 5 & 2 & -2 \end{pmatrix}$$

multiply 1st row by 5

$$= \begin{pmatrix} 5 & 35 & 5 \\ 5 & -9 & 0 \\ 5 & 2 & 2 \end{pmatrix}$$

Subtract the 1st row from the 2nd Row.

$$= \begin{pmatrix} 5 & 35 & 5 \\ 0 & -44 & -5 \\ 5 & 2 & 2 \end{pmatrix}$$

subtract the 1st row from 3rd Row.

$$= \begin{pmatrix} 1 & 7 & 1 \\ 0 & -44 & -5 \\ 0 & -33 & -3 \end{pmatrix}$$

Divide 2nd Row by −44.

$$= \begin{pmatrix} 1 & 7 & 1 \\ 0 & 1 & 5/44 \\ 0 & -33 & -3 \end{pmatrix}$$

multiply 2nd Row by −33.

$$= \begin{pmatrix} 1 & 7 & 1 \\ 0 & -33 & -15/4 \\ 0 & -33 & -3 \end{pmatrix}$$

Subtract 2nd Row from ~~third~~ 3rd Row.

$$= \begin{pmatrix} 1 & 7 & 1 \\ 0 & 1 & 5/44 \\ 0 & 0 & 3/4 \end{pmatrix}$$

return 2nd row to original value

$$= \begin{pmatrix} 1 & 7 & 1 \\ 0 & -44 & -5 \\ 0 & 0 & 3/4 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 7 & 1 \\ 0 & -44 & -5 \\ 0 & 0 & 3/4 \end{pmatrix}$$

$\therefore$ Rank of matrix $= 3$

number of non-zero rows.

3. $A = \begin{pmatrix} 1 & -3 & 6 \\ 4 & 0 & 2 \\ 8 & 5 & 1 \end{pmatrix}$ $B = \begin{pmatrix} 1 & 1 & 2 \\ 1 & -2 & 1 \\ 0 & 3 & -4 \end{pmatrix}$ $C = \begin{pmatrix} 0 & 4 & 3 \\ 6 & -7 & 1 \\ 1 & -3 & 2 \end{pmatrix}$

$$A = \begin{pmatrix} 1 & -3 & 6 \\ 4 & 0 & 2 \\ 8 & 5 & 1 \end{pmatrix}$$

$$|A| = 1\begin{vmatrix} 0 & 2 \\ 5 & 1 \end{vmatrix} + 3\begin{vmatrix} 4 & 2 \\ 8 & 1 \end{vmatrix} + 6\begin{vmatrix} 4 & 0 \\ 8 & 5 \end{vmatrix}$$

$$= 1(0+10) + 3(4-16) + 6(20-0)$$

$$= 1(10) + 3(-12) + 6(20)$$

$$= 10 - 36 + 120$$

$$= 94$$

$|A| \neq 0 \therefore A$ is a non-singular matrix

$$B = \begin{pmatrix} 1 & 1 & 2 \\ 1 & -2 & 1 \\ 0 & 3 & -4 \end{pmatrix}$$

$$|B| = \begin{vmatrix} 1 & 1 & 2 \\ 1 & -2 & 1 \\ 0 & 3 & -4 \end{vmatrix}$$

$$= 1\begin{vmatrix} -2 & 1 \\ 3 & -4 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 0 & -4 \end{vmatrix} + 2\begin{vmatrix} 1 & -2 \\ 0 & 3 \end{vmatrix}$$

$$= 1(8-3) - 1(-4-0) + 2(3-0)$$

$$= \cancel{1(5)-1(-5)+2(3)} \qquad 1(5)-1(-4)+2(3)$$

$$= \cancel{5+5+6} \qquad 5+4+6$$

$$= \cancel{+6} \qquad 15$$

$\therefore |B| \neq 0$ it's a non-singular matrix.

$$C = \begin{pmatrix} 0 & 4 & 3 \\ 6 & -7 & 1 \\ 1 & -3 & 2 \end{pmatrix}$$

$$|C| = \begin{vmatrix} 0 & 4 & 3 \\ 6 & -7 & 1 \\ 1 & -3 & 2 \end{vmatrix}$$

$$= 0\begin{vmatrix} -7 & 1 \\ -3 & 2 \end{vmatrix} - 4\begin{vmatrix} 6 & 1 \\ 1 & 2 \end{vmatrix} + 3\begin{vmatrix} 6 & -7 \\ 1 & -3 \end{vmatrix}$$

$$= 0(-14+3) - 4(12-1) + 3(-18+7)$$

$$= 0 + 44 - 33$$

$$= 11$$

$\therefore |C| \neq 0$ it's a non-singular matrix.