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17/Eng04/075

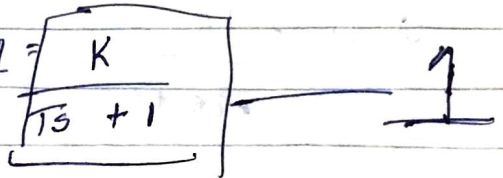
ELEC / ELECT

01

## SYSTEM RESPONSE 1

$$5) : G(s) = \frac{2}{0.2s + 0.5} \quad (2)$$

Equation  
from the ~~transfer function~~



$$G(s) = \frac{2}{0.2s + 0.5} = \frac{2/0.5}{\frac{0.2s + 0.5}{0.5}} = \frac{4}{0.4s + 1}$$

$$G(s) = \frac{4}{0.4s + 1}$$

from the equation 1, we have

$$\frac{4}{0.4s + 1} = \frac{K}{T_s + 1}$$

$$\boxed{(K) \text{ DC gain} = 4} , \quad \boxed{\text{Time constant } (T) = \frac{0.4}{0.4} = 1}$$

$$ii) G(s) = \frac{0.2}{0.05s + 0.1}$$

$$\frac{\frac{0.2}{0.1}}{\frac{0.05s + 0.1}{0.1}} = \frac{2}{0.5s + 1}$$

from equation 1 we have  $\boxed{K = 2}$  ,  $\boxed{T = 0.5}$

$$\text{iii) } G(s) = \frac{2}{3s+1}$$

The transfer function is already in order, so

$$\boxed{T=3}, \boxed{K=2}$$

$$\text{iv) } G(s) = \frac{16}{s^2+4s+4}$$

$$G(s) = \frac{16/4}{s^2/4 + s + 1} = \frac{4}{s^2/4 + s + 1}$$

$$\boxed{T=2}, \boxed{K=4}$$

### Question 4

$$G(s) = \frac{1}{3s+1} = \frac{1}{Ts+1}$$

$$\frac{R(s)}{F(s)} = \frac{1}{Ts+1}$$

$$R(s) = \frac{F(s)}{Ts+1}, \text{ The input is a ramp } F(s) = \frac{C}{s^2}$$

so ramp input =  $\frac{C}{s^2}$

$$R(s) = \frac{C}{s^2(Ts+1)}$$

$$\frac{C/T}{s^2(s+1/T)}$$

$$\left( \frac{Ka}{s^2(s+a)} \right) \text{ - frequency domain}$$

$$K \left( t - T \left( 1 - e^{-\frac{t}{T}} \right) \right) \text{ - Time domain}$$

$$x(t) = C \left( t - T \left( 1 - e^{-\frac{t}{T}} \right) \right)$$

$$= C (t - T (1))$$

$$= C (t - T)$$

$$= ct - cT$$

for steady state error we will assume  $(e^{-t/T})$  to be small

from  $\theta_e = \theta_i - \theta_o$

$$\theta_e = ct - (ct - cT)$$

$$\theta_e = cT$$

$$\theta_e = ct = 4 \times 3 = 12 \text{ mm}$$

$$\text{ii) } x(t) = 4 (2.3 (1 - 0.5))$$

$$= 4 (2.3 (0.49))$$

$$\text{ii) } x(t) = ct - (cT (1 - e^{-t/T}))$$

$$= 4 \times 2 - (4 \times 3 (1 - e^{-\frac{2}{3}}))$$

$$= 8 - (12 (1 - 0.5))$$

$$= 8 - (12 \times 0.49) = 8 - 5.88$$

steady state error = 2.161 mm

### Question 6

$$\frac{\omega}{\theta} (s) = \frac{K_m}{T_m s + 2}$$

where  $K_m = 15 \text{ s}^{-1}$   
 $T_m = 4 \text{ s}$

$$\frac{\omega(s)}{\theta} = \frac{15s^{-1}}{4s + 2} = \frac{15/2}{4/2 s + 2/2} = \frac{7.5}{2s + 1}$$

$$K = 7.5$$

$$T = 2 \text{ seconds}$$

1) From Diagram 1, we can deduce that

$$F(t) = Kx + B \frac{dx}{dt}$$

$$= \cancel{0.03x} + 4$$

$$= 0.03 \frac{dx}{dt} + 4000x$$

$$F(s) = 0.03(s) + 4000x$$

$$F(s) = x(0.03s + 4000)$$

$$\frac{x(s)}{F(s)} = \frac{1}{0.03s + 4000}$$

$$\frac{x(s)}{F(s)} = \frac{1/4000}{\frac{0.03s}{4000} + \frac{4000}{4000}}$$

$$= \frac{2.5 \times 10^{-4}}{(s) 7.5 \times 10^{-6} + 1}$$

Transfer function

$$\frac{-x(s)}{F(s)} = \frac{F(s) (2.5 \times 10^{-4})}{7.5 \times 10^{-6} s + 1}$$

$$= \frac{H (2.5 \times 10^{-4})}{s (7.5 \times 10^{-6} s + 1)}$$

$$\frac{K_a}{s(s+a)} = K \left[ 1 - e^{-t/\tau} \right]$$

$$= \frac{H (2.5 \times 10^{-4})}{7.5 \times 10^{-6}}$$

$$= \frac{H (3.33 \times 10^{-4})}{s(s + 1.3 \times 10^{-7})}$$

$$s \left( \frac{2.5 \times 10^{-4}}{7.5 \times 10^{-6}} + \frac{1}{7.5 \times 10^{-6}} \right)$$

$$s(s + 1.3 \times 10^{-7})$$

$$= \frac{H (2.5 \times 10^{-4})}{s(s + (1.3 \times 10^{-7}))}$$

M

from equation (M)

and from

$$\left[ \begin{array}{c} \frac{K}{T} \\ S(s + \gamma_T) \end{array} \right] - \textcircled{y}$$

$$\therefore \boxed{t = T = 7.5 \mu s}$$

from equ  $y$ , ~~the~~  $K \left( 1 - e^{-t/T} \right)$

$$K = 100 \times 2.5 \times 10^{-4} = 4 \times 10^{-3}$$

$$= 4 \times 10^{-3} \left( 1 - e^{-\frac{t}{T}} \right)$$

$$= 4 \times 10^{-3} (0.63)$$

$$= 2.52 \times 10^{-3}$$

# System Response 2

$$2) \frac{X_o(s)}{X_i} = \frac{1}{T^2 s^2 + 2\delta T s + 1}$$

$$\frac{Y_o(j\omega)}{X_i} = \frac{1}{T^2 (j\omega)^2 + 2\delta T j\omega + 1}$$

$$\frac{X_o(j\omega)}{X_i} = \frac{1}{(1 - T^2 \omega^2) + j 2\delta T \omega} \quad (\text{Note } j^2 = -1)$$

$$\frac{X_o(j\omega)}{X_i} = \frac{1}{A + jB}$$

where  $A = (1 - T^2 \omega^2)$  and  $B = 2\delta T \omega$

$$\frac{X_o(j\omega)}{X_i} = \frac{1 \times A - jB}{(A + jB)(A - jB)} = \frac{A - jB}{A^2 + jBA - jBA - jB^2}$$

$$= \frac{A - jB}{A^2 + B^2} \quad \text{--- } \textcircled{X}$$

from equ  $\textcircled{X}$

$$C = \frac{A}{A^2 + B^2} = \frac{(1 - \omega^2 T^2)}{(1 - \omega^2 T^2)^2 + (2\delta \omega T)^2}$$

$$D = \frac{B}{A^2 + B^2} = \frac{2\delta T \omega}{(1 - T^2 \omega^2)^2 + (2\delta \omega T)^2}$$

Solving for A & B

$$A = 1 - T^2 \omega^2$$

$$= 1 - (0.4^2 \times 2.5^2)$$

$$A = 0$$

$$B = 2T\delta\omega$$

$$B = 2 \times 0.4 \times 0.2 \times 2.5$$

$$B = 0.4$$

$$C = \frac{0}{0 + 0.16} = 0$$

$$D = \frac{0.4}{0.16} = 2.5$$

$$\phi = -\tan^{-1}\left(\frac{2.5}{0}\right)$$

$$= -\tan^{-1}(\infty)$$

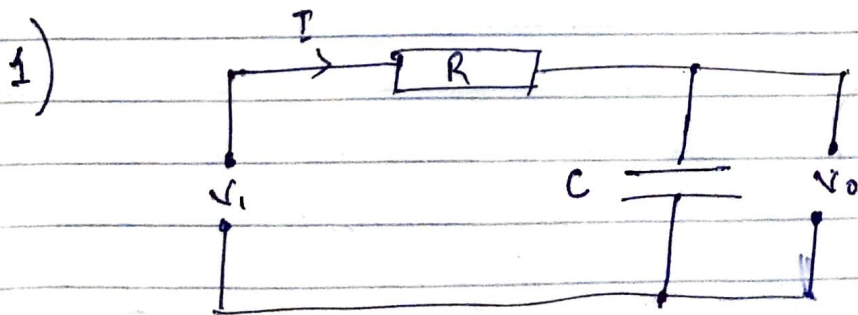
$$= -90^\circ$$

$$R = \sqrt{C^2 + D^2} = \sqrt{0^2 + 2.5^2}$$

$$R = 2.5$$

$$\text{Amplitude} = 2.6 \times 6 = 16$$

The ~~Output equation~~ :  ~~$\theta_1 = 16 \sin(\omega t)$~~



$$\frac{V_o}{V_i}(s) = \frac{1}{Ts + 1}$$

$$\frac{V_o}{V_i}(j\omega) = \frac{1}{j\omega T + 1}$$

$$\frac{V_o}{V_{in}} = \frac{1}{(1+j\omega T)(1-j\omega T)} = \frac{1}{1-\omega^2 T^2}$$

(j's cancel out when  $j^2 = -1$ )

$$= \frac{1}{1-\omega^2 T^2} - \frac{j\omega T}{1-\omega^2 T^2}$$

$$\frac{V_o}{V_{in}} (j\omega) = A - jB$$

$A = \frac{1}{1+\omega^2 T^2}$

$B = \frac{\omega T}{1+\omega^2 T^2}$

$$A = \frac{1}{1 + (2000 \times 9.4 \times 10^{-4})^2}$$

Note:  $T = RC = 47 \times 20 \mu F$

$$T = 9.4 \times 10^{-3}$$

$$T = 9.4 \times 10^{-4}$$

$$A = 0.22$$

$$B = \frac{2000 \times 9.4 \times 10^{-4}}{1 + (2000 \times 9.4 \times 10^{-4})^2} = \frac{1.88}{1 + 3.53}$$

$$B = 0.41$$

$$\theta = \tan^{-1} \left( \frac{B}{A} \right)$$

$$= \tan^{-1} \left( \frac{0.41}{0.22} \right)$$

$$\theta = 61.78^\circ$$

$$R = \sqrt{A^2 + B^2} = \sqrt{0.22^2 + 0.41^2} = \sqrt{0.048 + 0.168}$$

$$R = 0.465$$

$$\text{Amplitude} = 5 \times 0.465 = 2.33$$



From Q. Vm equation  $\rightarrow 5 \sin(2000t)$

$$V_{out} = 2.33 \sin(2000t - 61.78^\circ)$$

