

$$\textcircled{1} \int \frac{3x-1}{(x-3)(x-2)(x-1)} dx$$

$$= \int \left( \frac{1}{x-1} - \frac{5}{x-2} + \frac{4}{x-3} \right) dx$$

$$= \int \frac{1}{x-1} dx - 5 \int \frac{1}{x-2} dx + 4 \int \frac{1}{x-3} dx$$

$$= \ln(x-1) - 5 \ln(x-2) + 4 \ln(x-3) + C$$

$$\textcircled{2} \int \frac{(x^2+x+1)}{(x^2)(x^2+1)} dx$$

$$= \frac{1}{5} \int \left( \frac{2x+1}{5(x^2+1)} + \frac{3}{5(x+2)} \right) dx$$

$$= \frac{1}{5} \int \frac{2x+1}{x^2+1} dx + \frac{3}{5} \int \frac{1}{x+2} dx$$

$$= \frac{1}{5} \int \left( \frac{2x}{x^2+1} + \frac{1}{x^2+1} \right) dx + \frac{3}{5} \ln(x+2)$$

$$= \frac{1}{5} \left[ 2 \int \frac{x}{x^2+1} dx + \int \frac{1}{x^2+1} dx \right] + \frac{3}{5} \ln(x+2)$$

$$u = x^2 + 1 \quad \frac{du}{dx} = 2x \quad dx = \frac{1}{2x} du$$

$$= \frac{1}{2} \int \frac{1}{u} du$$

$$= \frac{1}{2} \ln u \quad \therefore = \frac{1}{2} \ln(x^2 + 1)$$

$$\int \frac{1}{x^2+1} = \tan^{-1}(x)$$

$$= \frac{1}{5} \left[ 2 \left( \frac{1}{2} \ln(x^2+1) \right) + \tan^{-1}(x) \right] + \frac{3}{5} \ln(x+2) + C$$

$$= \frac{3 \ln(x+2) + \ln(x^2+1) + \tan^{-1}(x)}{5} + C$$

=

$$(3) \int \frac{x^2+1}{(x-3)(x-2)^2} dx$$

$$= \int \left( \frac{10}{x-3} - \frac{9}{x-2} - \frac{5}{(x-2)^2} \right) dx$$

$$= 10 \int \frac{1}{x-3} - 9 \int \frac{1}{x-2} - 5 \int \frac{1}{x-2}$$

$$u = x-2 \quad \frac{du}{dx} = 1 \quad du = dx$$

$$\int \frac{1}{u} du = \ln u ; \int \frac{1}{u^2} du = -\frac{1}{u}$$

$$= 10 \int \frac{1}{x-3} - 9 \ln(x-2) - 5 \left( -\frac{1}{x-2} \right)$$

$$= 10 \int \frac{1}{x-3} - 9 \ln(x-2) + \frac{5}{x-2}$$

$$u = x-3 \quad \frac{du}{dx} = 1 \quad \int \frac{1}{u} du = \ln u$$

$$= 10 \ln(x-3) - 9 \ln(x-2) + \frac{5}{x-2} + C$$

$$(4) \int \frac{x^2+x^2+x+1}{x-1}$$

$$u = x-1 \quad \therefore x = u+1 \quad \frac{du}{dx} = 1 \quad dx = du$$

$$x^2 = (u+1)^2$$

$$x^3 = (u+1)^3$$

$$\therefore = \int \frac{u^3 + 4u^2 + 6u + 4}{u} du$$

$$= \int u^2 + 4u + 6 + \frac{4}{u} du$$

$$= \frac{u^3}{3} + \frac{4u^2}{2} + 6u + 4 \ln u + C$$

$$= \frac{(x-1)^3}{3} + \frac{4(x-1)^2}{2} + 6(x-1) + 4 \ln(x-1) + C$$

$$= \frac{(x-1)^3}{3} + 2(x-1)^2 + 6(x-1) + 4 \ln(x-1) + C$$