14/ENG05/031

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MAT 102

(1) $M = pi - 6j - 3k \quad N = 4i + 3j - k \quad O = i - 3j + 2k$

$M \cdot N = 0$

$4p - 6(3) - 3(-1) = 0$

$4p - 18 + 3 = 0$

$4p = 18 - 3$

$4p = 15$

$p = \frac{15}{4}$

$M \cdot (N \times O) = 0$

$$\begin{vmatrix} p & -6 & -3 \\ 4 & 3 & -1 \\ 1 & -3 & 2 \end{vmatrix} = p(6-3) + 6(8+1) - 3(-12-3) = 0$$

$3p + 54 + 45 = 0$

$3p + 99 = 0$

$3p = -99$

$p = -33$

(2) $3i + 2j + 5k \quad 2i - j + 6k \quad 5i + 2j - 3k$

$\vec{r} = 10i + 3j + 8k$

$\hat{r} = \frac{r}{|r|}$

$|r| = \sqrt{10^2 + 3^2 + 8^2} = \sqrt{100 + 9 + 64} = \sqrt{173}$

$\hat{r} = \frac{10i + 3j + 8k}{\sqrt{173}}$

$\cos A = \frac{10}{\sqrt{173}}$

$\cos B = \frac{3}{\sqrt{173}} \quad \cos C = \frac{8}{\sqrt{173}}$

③ $F = 3xi + y^2 j + (x+z)k$

$N = xi - 3y j + (y-z)k$

$F \times N = \begin{vmatrix} i & j & k \\ 3x & y^2 & x+z \\ x & -3y & y-z \end{vmatrix}$

$= i((y^3 - 2y^2) + (3xy^2 + 6xy)) - j(3xy^2 - 6xy - 4xy^2 - 4xy)$

$+ k(-9xy^2 - 2xy^3)$

$= (y^3 + xy^2 + 6xy)i + (xy^2 + 10xy)j + (9xy^2 + 2xy^3)k$

$\int_0^1 (y^3 + xy^2 + 6xy)i + (xy^2 + 10xy)j + (2xy^3 + 9xy^2)k$

$= i \int_0^1 y^3 + xy^2 + 6xy + j \int_0^1 xy^2 + 10xy - k \int_0^1 2xy^3 + 9xy^2$

$= i\left(\frac{y^4}{4} + \frac{xy^3}{3} + \frac{6xy^2}{2}\right)_0^1 + j\left(\frac{xy^3}{3} + \frac{10xy^2}{2}\right)_0^1 - k\left(\frac{2xy^4}{4} + \frac{9xy^3}{3}\right)_0^1$

$= i\left(\frac{1}{4} + \frac{1}{3} + 3\right) + j\left(\frac{1}{3} + 5\right) - k\left(\frac{1}{2} + 3\right)$

$= \frac{43}{12}i + \frac{16}{3}j - \frac{7}{2}k$