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Questions

Find the integral of the following (i) $\frac{(3x-1)}{(x-1)(x-2)(x-3)}dx$ (ii) $\frac{(x^2+x+1)}{(x+2)(x^2+1)}$
(iii) $\frac{(x^2+1)}{(x-3)(x-2)^2}dx$ (iv) $\frac{(x^3+x^2+x+1)}{(x-1)}dx$

Solution

1) $\frac{(3x-1)}{(x-1)(x-2)(x-3)} = \frac{A}{(x-1)} + \frac{B}{(x-2)} + \frac{C}{(x-3)}$

$= \frac{A(x-2)(x-3)+B(x-1)(x-3)+C(x-1)(x-2)}{(x-1)(x-2)(x-3)}$

Cancelling denominator

$3x-1 = A(x-2)(x-3)+B(x-1)(x-3)+C(x-1)(x-2)$

let $x=1$

$3(1)-1 = A(1-2)(1-3)+B(1-1)(1-3)+C(1-1)(1-2)$

$2 = A(-1)(-2)+B\times0+C\times0$

$2 = 2A \quad \therefore A = 1$

let $x=2$

$3(2)-1 = A(2-2)(2-3)+B(2-1)(2-3)+C(2-1)(2-2)$

$6-1 = A\times0+B(1)(-1)+C\times0$

$5 = -1 \quad \therefore B = -5$

let $x=3$

$3(3)-1 = A(3-2)(3-3)+B(3-1)(3-3)+C(3-1)(3-2)$

$9-1 = A\times0+B\times0+C\times(2)(1)$

$8 = 2C \quad \therefore C=4$

Hence

$\int\frac{3x-1}{(x-1)(x-2)(x-3)} = \int\frac{1}{x-1} + \frac{-5}{x-2} + \frac{4}{x-3}dx$

$= \int\frac{1}{x-1}dx - 5\int\frac{1}{x-2}dx + 4\int\frac{1}{x-3}dx$

$= \log|x-1| - 5\log|x-2| + 4\log|x-3| + C$

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2) $\dfrac{(x^2+x+1)}{(x+2)(x^2+1)}dx = \dfrac{x^2+x+1}{(x+1)(x+2)} = \dfrac{A}{x+2} + \dfrac{Bx+C}{x^2+1}$

Cancelling denominator

$x^2+x+1 = A(x^2+1) + (Bx+C)(x+2)$

let $x = -2$

$(-2)^2 + (-2) + 1 = A((-2)^2+1) + 0$

$4 - 2 + 1 = 5A \qquad 3/5 = A$

let $x = 0$

$x^2+x+1 = A(x^2+1) + (Bx+C)(x+2)$

$0^2 + 0 + 0 = A(0+1) + (0+C)(0+2)$

$1 = A + 2C$

$1 = 3/5 + 2C$

$1 - \dfrac{3}{5} = 2C$

$2/5 = 2C \qquad C = 1/5$

let $x = 1$

$x^2+x+1 = A(x^2+1) + (Bx+C)(x+2)$

$1^2+1+1 = 2A + (B+C)(3)$

$3 = 2A + 3(B+C)$

$3 = 2(3/5) + 3(B + 1/5)$

$3 - \dfrac{6}{5} = 3\left(B = \dfrac{1}{5}\right)$

$\dfrac{9}{5} = 3\left(B + \dfrac{1}{5}\right)$

$3/5 - \dfrac{1}{5} = B \quad \therefore B = 2/5$

$\dfrac{x^2+x+1}{(x+1)(x+2)} = \dfrac{A}{x+2} + \dfrac{Bx+C}{x^2+1}$

$\dfrac{x^2+x+1}{(x+1)(x^2+1)} = \dfrac{3}{5(x+2)} + \dfrac{1(2x+1)}{5(x^2+1)}$

$\displaystyle\int \dfrac{x^2+x+1}{(x+2)(x^2+1)}dx = \int \dfrac{3}{5(x^2+1)}dx + \int \dfrac{1}{5}\,\dfrac{(2x+1)}{x^2+1}dx$

$$= \int \frac{3}{5(x^2+1)}\,dx + \frac{1}{5}\int \frac{2x}{x^2+1}\,dx + \frac{1}{5}\int \frac{1}{x^2+1}\,dx$$

equation i　　equation (ii)　　equation (iii)

$$\frac{3}{5}\int \frac{1}{x+2}\,dx$$

$$= \frac{3}{5}\log|x+2| + C_1$$

$$\frac{1}{5}\int \frac{2x}{x^2+1}\,dx$$

let $t = x^2+1$

$$\frac{dt}{dx} = 2x$$

$$dt = 2x\,dx$$

substituting

$$= \frac{1}{5}\int \frac{dt}{t}$$

$$= \frac{1}{5}\log|t| + C$$

$$= \frac{1}{5}\log(x^2+1) + C_2$$

$$\frac{1}{5}\int \frac{1}{x^2+1}\,dx$$

$$= \frac{1}{5}\tan^{-1}(x) + C_3$$

Hence

$$\int \frac{x^2+x+1}{(x+2)(x^2+1)}\,dx = \frac{3}{5}\log(x+2) + \frac{1}{5}\log(x^2+1) + \frac{1}{5}\tan^{-1}(x) + C$$

where $C = C_1 + C_2 + C_3$

3) $\frac{(x^2+1)}{(x-3)(x-2)^2}\,dx$　$\frac{x^2+1}{(x-3)(x-2)^2}\,dx$　$\frac{x+1}{(x-3)(x-2)}$　$\frac{x+1}{(x-3)(x-2)}$

$$\int \frac{\cancel{x^2(x+x)}}{\cancel{x}} \quad \frac{A}{(x-3)} + \frac{B}{(x-2)^2} = \frac{A(x-2)^2 + B(x-3)}{(x-3)(x-2)^2}$$

$$A(x-2)^2 + B(x-3)$$

let $x=1$

$$A(1-2)^2 + B(1-3)$$

$$A(1) + B(-2) \qquad \therefore A = -2$$

let $x=2$

$$A(2-2)^2 + B(2-3)$$

$$A(0) + B(-1) \qquad \therefore B = 0$$

4) $\dfrac{(x^3+x^2+x+1)}{(x-1)}\,dx$

$$\int \frac{x^3+x^2+x+1}{(x-1)}\,dx$$

$$\int \frac{x^2(x+1)+1(x+1)}{x-1}\,dx$$

$$\int \frac{(x^2+1)(x+1)}{x-1}\,dx$$

$$\int (x^2+1)\,dx$$

$$\int (x^2+x^0)\,dx$$

$$\int x^2\,dx+\int x^0\,dx$$

$$=\frac{x^{2+1}}{2+1}+\frac{x^{0+1}}{0+1}+C$$

$$=\frac{x^3}{3}+x+C$$