Name: Effanga, Bassey Effanga
19/sci01/041
MAT 104

① $\int \frac{(3x-1)}{(x-1)(x-2)(x-3)}$

$\frac{(3x-1)}{(x-1)(x-2)(x-3)} = \frac{A}{(x-1)} + \frac{B}{(x-2)} + \frac{C}{(x-3)}$

Multiply through by $(x-1)(x-2)(x-3)$

$(3x-1) = (x-2)(x-3)A + (x-1)(x-3)B + (x-1)(x-2)C$

$(3x-1) = Ax^2 - 5Ax$

$\frac{(3x-1)}{(x-1)(x-2)(x-3)} = \frac{A}{(x-1)} + \frac{B}{(x-2)} + \frac{C}{(x-3)}$

$\frac{A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)}{(x-1)(x-2)(x-3)}$

Equate the numerator of the R.H.S to the numerator of the L.H.S

$(3x-1) = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)$

let $x = 1$

$3(1) - 1 = A(1-2)(1-3) + B(1-1)(1-3) + C(1-1)(1-2)$

$2 = A(-1)(-2) + B(0)(-2) + C(0)(-1)$

$\frac{2}{2} = \frac{2A}{2}$

$A = 1$

let $x = 2$

$3(2) - 1 = A(2-2)(2-3) + B(2-1)(2-3) + C(2-1)(2-2)$

$5 = A(0)(-1) + B(1)(-1) + C(1)(0)$

$\frac{5}{-1} = \frac{-B}{-1}$

$B = -5$

let $x = 3$

$3(3) - 1 = A(3-2)(3-3) + B(3-1)(3-3) + C(3-1)(3-2)$

$8 = A(1)(0) + B(2)(0) + C(2)(1)$

$\frac{8}{2} = \frac{2C}{2}$

$C = 4$

∴ The resolve is

$\frac{(3x-1)}{(x-1)(x-2)(x-3)} = \frac{1}{(x-1)} + \frac{-5}{(x-2)} + \frac{4}{(x-3)}$

$\int \frac{(3x-1)}{(x-1)(x-2)(x-3)} dx \Rightarrow$

$\int \frac{1}{(x-1)} dx - \int \frac{-5}{(x-2)} dx + \int \frac{4}{x-3} dx$

$\Rightarrow \ln(x-1) - 5\int \frac{1}{(x-2)} dx + 4\int \frac{1}{x-3} dx$

$\Rightarrow \ln(x-1) - 5\ln(x-2) + 4\ln(x-3) + C$

② $\int \frac{x^2 + x + 1}{(x+2)(x^2+1)} dx$

$\frac{x^2 + x + 1}{\cancel{(x+2)}(x^2+1)} = \frac{A}{(x+2)} + \frac{Bx + C}{(x^2+1)}$

$\frac{A(x^2+1) + [(Bx+C)(x+2)]}{\cancel{(x+2)(x^2+1)}}$

$x^2 + x + 1 = A(x^2+1) + [(Bx+C)(x+2)]$

let $x = -2$

$(-2)^2 + (-2) + 1 = A(-2)^2 + 1) + [(B(-2)+C)(-2+2)$

$4 + (-2) + 1 = 5A + 0$

$\frac{3}{5} = \frac{5A}{5}$

$A = 3/5$

from R.H.S

$A(x^2+1) + [(Bx+C)(x+2)]$

$= Ax^2 + A + [Bx^2 + 2Bx + Cx + 2C]$

$= Ax^2 + A + Bx^2 + 2Bx + Cx + 2C$

Collect like terms

$= Ax^2 + Bx^2 + 2Bx + Cx + A + 2C$

$= x^2(A+B) + x(2B+C) + A + 2C$

$x^2 + x + 1 = x^2(A+B) + x(2B+C) + A + 2C$

Compare the co-efficient

$1 = A + B$ —— (i)

$1 = 2B + C$ —— (ii)

$1 = A + 2C$ —— (iii)

Substitute $A = \frac{3}{5}$ in —— (ii)

$1 = \frac{3}{5} + B$

$B = 1 - \frac{3}{5}$

$B = \frac{2}{5}$

Substitute $B = \frac{2}{5}$ in —— (ii)

$1 = 2\left(\frac{2}{5}\right) + C$

$1 = \frac{4}{5} + C$

$C = 1 - \frac{4}{5} = \frac{1}{5}$

∴ The resolve is

$$\frac{x^2+x+1}{(x+2)(x^2+1)} = \frac{3}{5(x+2)} + \frac{2x+1}{5(x^2+1)}$$

$$\Rightarrow \int \frac{x^2+x+1}{(x+2)(x^2+1)}\,dx = \int \frac{3}{5(x+2)}\,dx + \int \frac{2x+1}{(5(x^2+1))}\,dx$$

$$= \int \frac{3}{5(x+2)}\,dx + \frac{1}{5}\int \frac{2x+1}{(x^2+1)}\,dx$$

$$= \frac{3}{5}\int \frac{1}{(x+2)}\,dx + \frac{1}{5}\left[\int \frac{2x}{(x^2+1)}\,dx + \int \frac{1}{(x^2+1)}\,dx\right]$$

$$= \frac{3}{5}\ln(x+2) + \frac{1}{5}\left[\int \frac{2x}{4} \times \frac{1}{2x}\,du + \arctan(x)\right]$$

$$= \frac{3}{5}\ln(x+2) + \frac{1}{5}\left(\int \frac{1}{4}\,du + \arctan(x)\right)$$

$$= \frac{3}{5}\ln(x+2) + \frac{1}{5}\left\{\ln(x^2+1) + \arctan(x)\right\} + C$$

$$\therefore \int \frac{x^2+x+1}{(x+2)(x^2+1)}\,dx = \frac{3}{5}\ln(x+2) + \frac{1}{5}\ln(x^2+1) + \frac{\arctan(x)}{5} + C$$

(3)

$$\int \frac{(x^2+1)}{(x-3)(x-2)^2}\,dx$$

$$\frac{(x^2+1)}{(x-3)(x-2)^2} = \frac{x^2+1}{(x-3)(x^2-4x+4)}$$

$$\frac{(x^2+1)}{\cancel{(x-3)(x^2-4x+4)}} = \frac{A}{(x-3)} + \frac{Bx+C}{(x^2-4x+4)}$$

$$= \frac{A(x^2-4x+4) + [(Bx+C)(x-3)]}{(x-3)(x^2-4x+4)}$$

$(x^2+1) = A(x^2-4x+4) + [(Bx+C)(x-3)]$

let $x = 3$

$(3)^2+1 = A((3)^2 - 4(3) + 4) + [(B(3)+C)(3-3)]$

$10 = A(1)$

$A = 10$

from R.H.S

$A(x^2-4x+4) + [(Bx+C)(x-3)]$

$Ax^2 - 4Ax + 4A + [Bx^2 - 3Bx + Cx - 3C]$

$Ax^2 - 4Ax + 4A + Bx^2 - 3Bx + Cx - 3C$

Collect like term

$= Ax^2 + Bx^2 - 4Ax - 3Bx + Cx + 4A - 3C$

$= x^2(A+B) + x(-4A - 3B + C) + 4A - 3C$

$(x^2+1) = x^2(A+B) + x(-4A - 3B + C) + 4A - 3C$

Compare the co-efficient

$1 = A + B$ —— (i)

$0 = -4A - 3B + C$ —— (ii)

$1 = 4A - 3C$ —— (iii)

Substitute $A = 10$ in —— (ii)

$\cancel{0 = -4(10)}$ $1 = A + B$

$1 = 10 + B$

$B = 1 - 10$

$B = -9$

Substitute $B = -9$ and $A = 10$ in —— (3)

$0 = -4A - 3B + C$

$0 = -4(10) - 3(-9) + C$

$0 = -40 + 27 + C$

$0 = -13 + C$

$C = 13$

The resolve is $\frac{(x^2+1)}{(x-3)(x-2)^2} = \frac{10}{(x-3)} - \frac{9x+13}{(x^2-4x+4)}$

$\Rightarrow \int \frac{x^2+1}{(x-3)(x-2)^2}\,dx = \int \frac{10}{(x-3)}\,dx - \int \frac{9x+13}{(x^2-4x+4)}\,dx$

$= 10\ln(x-3) - 9\ln(x-2) + \frac{18}{(x-2)} - \frac{13}{(x-2)}$

$= 10\ln(x-3) - 9\ln(x-2) + \frac{18-13}{(x-2)}$

$= 10\ln(x-3) - 9\ln(x-2) + \frac{5}{(x-2)} + c$

$\therefore \int \frac{x^2+1}{(x-3)(x-2)^2}\,dx \Rightarrow$

$10\ln(x-3) - 9\ln(x-2) + \frac{5}{(x-2)} + c$

4

$\frac{(x^3+x^2+x+1)\,dx}{(x-1)}$

$\int \frac{x^3+x^2+x+1}{(x-1)}\,dx \Rightarrow$

$\int \frac{x^3}{(x-1)}\,dx + \int \frac{x^2}{(x-1)}\,dx + \int \frac{x}{(x-1)}\,dx + \int \frac{1}{(x-1)}\,dx$

$= \frac{2x^3+3x^2+6x-4}{6} + \ln(x-1) + \frac{x^2}{2} + x + \ln(x-1) + \ln(x-1) + \ln(x-1) + \ln(x-1) + c$

$= 2x^3+3x^2+6x-4 + \frac{x^2}{2} + x + x - 1 + \ln(x-1) + \ln(x-1) + \ln(x-1) + \ln(x-1) + c$

$\Rightarrow \frac{2x^3+3x^2+6x-11+3x^2+6x+6x-6}{6} + 4\ln(x-1) + c$

$\Rightarrow \frac{2x^3+6x^2+18x-17}{6} + 4\ln(x-1) + c$

4

$\therefore \int \frac{x^3+x^2+x+1}{(x-1)}\,dx \Rightarrow$

$\frac{2x^3+6x^2+18x-17}{6} + 4\ln(x-1) + c$