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Solution

1) $\frac{(3x-1)}{(x-1)(x-2)(x-3)} = \frac{A}{(x-1)} + \frac{B}{(x-2)} + \frac{C}{(x-3)}$

$3x-1 = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)$

$= A(x^2-5x+6) + B(x^2-4x+3) + C(x^2-3x+2)$

$= Ax^2 - 5Ax + 6A + Bx^2 - 4Bx + 3B + Cx^2 - 3Cx + 2C$

$0 = A + B + C$ — (i)

$3 = -5A - 4B - 3C$ (ii)

$-1 = 6A + 3B + 2C$ (iii)

from eqn (i)

$A = -B - C$

Substitute into eqn (ii) & (iii)

$3 = -5(-B-C) - 4B - 3C$

$-1 = 6(-B-C) + 3B + 2C$

$3 = 5B + 5C - 4B - 3C$ —— (iv)

$-1 = -6B - 6C + 3B + 2C$ —— (v)

$3 = B + 2C$ —— (vi)

$-1 = -3B - 4C$ (vii)

from eqn (vi)

$3 - 2C = B$ —— (viii)

Substitute into eqn (vii)

$-1 = -3(3-2C) - 4C$

$-1 = -9 + 6C - 4C$

$8 = 2C$

$4 = C$

Substitute $4 = C$ in eqn (viii)

$3 - 2(4) = B = -5$

$\therefore B = -5, \ C = 4$

substitute into $A = -B - C$

$A = -(-5) - 4$

$A = 1$

$\therefore A = 1, B = -5, C = 4$

$$\frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{1}{(x-1)} + \frac{-5}{(x-2)} + \frac{4}{(x-3)}$$

$$= \ln(x-1) - 5\ln(x-2) + 4\ln(x-3) + C$$

2) $$\frac{(x^2+x+1)}{(x+2)(x^2+1)}dx = \cancel{\frac{A}{(x+2)}} + \cancel{\frac{B}{(x+1)}} + \cancel{\frac{C}{(x+1)}} = \frac{A}{(x+2)} + \frac{Bx+C}{x^2+1}$$

$\cancel{x^2+x+1) = A(x+1)(x+1) + Bx}$

$x^2 + x + 1 = A(x^2+1) + Bx + C(x+2)$

$x^2 + x + 1 = Ax^2 + A + Bx^2 + 2Bx + Cx + 2C$

$1 = A + B$

$1 = 2B + C$

$1 = A + 2C$

$x^2 + x + 1 = A(x^2+1) + Bx + C(x+2)$

when $x = -2$

$4 - 2 + 1 = A(4+1) + 0$

$\frac{3}{5} = A$

when $x = 0$

$0 + 0 + 1 = A(0+1) + 0 + C(0+2)$

$1 = A + 2C$

$1 = A + 2C$

$\frac{1-A}{2} = 1 - A = 2C$

$\frac{1}{1} - \frac{3}{5} = 2C$

$\frac{\frac{2}{5}}{2} = C$

$= \frac{1}{5} = C$

from eqn when $x = 1$

$1+1+1 = A(1+1) + B + C(3)$

$3 = 2A + B + C(3)$

$3 = 2\frac{3}{5} + B + \frac{1}{5}(3)$

$3 - \frac{6}{5} = B + \frac{1}{5}(3)$

$\frac{9}{5} = B + \frac{1}{5}B$

$\frac{3}{5} - \frac{1}{5} = B$

$\frac{2}{5} = B$

$A, B, C = \frac{3}{5}, \frac{2}{5}, \frac{1}{5}$

$$\frac{x^2+x+1)}{(x+2)(x^2+1)} = \int \frac{\frac{3}{5}}{x+2)} dx + \int \frac{\frac{2}{5}x}{x^2+1} dx + \int \frac{\frac{1}{5}}{x^2+1} dx$$

$$\frac{3}{5}\int \frac{dx}{x+2} + \frac{1}{5}\int \frac{2x\,dx}{x^2+1} + \frac{1}{5}\int \frac{1}{x^2+1} dx$$

let $u = x^2+1$

$\frac{du}{dx} = 2x$

$= du = dx$

$$= \frac{3}{5}\int \frac{dx}{x+2} + \frac{1}{5}\int \frac{du}{(x^2+1)} = \frac{1}{5}\int \frac{dx}{x^2+1}$$

$$\frac{3}{5}\ln(x+2) + \frac{1}{5}\ln(x^2+1) + \frac{1}{5}\tan^{-1}(x) + c$$

$$\Rightarrow \frac{x^2+1}{(x-3)(x-2)^2} = \frac{A}{x-3} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$$

$$= A(x-2)^2 + B(x-3)(x-2) + C(x-3)$$

$$= Ax^2 - 4Ax + 4A + Bx^2 - 5Bx + 6B + Cx - 3C.$$

$1 = A + B$

$0 = -4A - 5B + C$

$1 = 4A + 6B - 3C$

$1 - A = B$

Substitute in eqn ii & iii

$0 = -4A - 5(1-A) + C$

$1 = 4A + 6(1-A) - 3C$

$0 = -4A - 5 + 5A + C$ — (iv)

$1 = 4A + 6 - 6A - 3C$ — (v)

$5 = A + C$ (vi)

$-5 = -2A - 3C$ (vii)

from eqn (vi)

$5 - A = C$ — (viii)

$-5 = 2A - 3(5-A)$

$-5 = 2A - 15 + 3A$

$15 - 5 = A$

$A = 10$

Substitute $A = 10$ in eqn (viii)

$5 - 10 = -5$

Substitute $C = -5$, $A = 10$ in eqn (ii)

$0 = -4(10) - 5(B) + (-5)$

$0 = -40 - 5B - 5$

$B = -9$

$A, B, C = 10, -9, -5$

$$\frac{x^2-1}{(x-3)(x-2)^2} = \frac{A}{(x-3)} + \frac{B}{(x-2)} + \frac{C}{(x-3)^2}$$

$$= \int\frac{10}{x-3} - \int\frac{9}{x-2} + \int\frac{5}{(x-2)^2}$$

$$= 10\ln(x-3) - 9\ln(x-2) - \frac{5}{(x-2)} + c$$

$$10\ln(x-3) - 9\ln(x-2) + 5(x-2)^{-1} + c$$

4) $\dfrac{x^3+x^2+x+1}{x-1}\,dx$

using long division

$$\begin{array}{r|l} & x^2+2x+3 \\ \hline x-1 & x^3+x^2+x+1 \\ & -x^3+x^2 \\ \hline & +2x^2+x+1 \\ & -2x^2+2x \\ \hline & 3x+1 \\ & -3x+3 \\ \hline & 4 \end{array}$$

$$\therefore \int x^2+2x+3 \int\frac{4}{x-1}$$

$$\int x^2 + \int 2x + \int 3 + \int\frac{4}{x-1}$$

$$\cancel{\int\frac{x^3}{3} + \int x^2 + \int 3x} +$$

$$\frac{x^3}{3} + x^2 + 3x + 4\ln(x-1)$$