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MAT104

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$$\int \frac{(3x-1)}{(x-1)(x-2)(x-3)} dx$$

$$\frac{(3x-1)}{(x-1)(x-2)(x-3)} = \frac{A}{(x-1)} + \frac{B}{(x-2)} + \frac{C}{(x-3)}$$

$$\frac{(3x-1)}{(x-1)(x-2)(x-3)} = \frac{A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)}{(x-1)(x-2)(x-3)}$$

$$3x-1 = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)$$

when  $x=1$

$$3(1)-1 = A(1-2)(1-3)$$

$$3-1 = A(-1)(-2)$$

$$2 = A(2)$$

$$A = 1$$

when  $x=2$

$$3(2)-1 = B(2-1)(2-3)$$

$$6-1 = B(1)(-1)$$

$$5 = -1B$$

$$B = -5$$

when  $x=3$

$$3(3)-1 = C(3-1)(3-2)$$

$$9-1 = C(2)(1)$$

$$8 = 2C$$

$$C = 4$$

$$\int \frac{3x-1}{(x-1)(x-2)(x-3)} = \int \frac{1}{(x-1)} dx + \int \frac{-5}{(x-2)} dx + \int \frac{4}{(x-3)} dx$$

$$= \ln|x-1| - 5 \ln|x-2| + 4 \ln|x-3| + C$$



$$2 \int \frac{x^2 + x + 1}{(x+2)(x^2+1)}$$

$$\frac{x^2 + x + 1}{(x+2)(x^2+1)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+1}$$

$$\frac{x^2 + x + 1}{(x+2)(x^2+1)} = \frac{A(x^2+1)}{(x+2)(x^2+1)} + \frac{(Bx+C)(x+2)}{(x+2)(x^2+1)}$$

$$x^2 + x + 1 = A(x^2+1) + (Bx+C)(x+2)$$

When  $x = -2$

$$2^2 + (-2) + 1 = A(2^2 + 1)$$

$$4 - 2 + 1 = A(4 + 1) = 5A$$

$$3 = 5A \quad A = \frac{3}{5}$$

expand the whole R.H.S

$$A(x^2+1) + Bx(x+2) + C(x+2)$$

$$(Ax^2 + Bx^2) + (2Bx + Cx) + (A + 2C)$$

$$x^2(A+B) + x(2B+C) + (A+2C)$$

(compare R.H.S to L.H.S)

$$A + B = 1$$

$$\frac{3}{5} + B = 1$$

$$B = 1 - \frac{3}{5} = \frac{2}{5}$$

$$2B + C = 1$$

$$2\left(\frac{2}{5}\right) + C = 1$$

$$\frac{4}{5} + C = 1$$

$$C = 1 - \frac{4}{5} = \frac{1}{5}$$

$$\int \frac{x^2 + x + 1}{(x+2)(x^2+1)} = \int \frac{\frac{3}{5}}{x+2} + \frac{\frac{2}{5}x + \frac{1}{5}}{x^2+1} dx$$

$$\frac{3}{5} \int \frac{1}{x+2} + \frac{1}{5} \int \frac{2x}{x^2+1} + \frac{1}{5} \int \frac{1}{x^2+1} dx$$

$$\frac{3}{5} \ln|x+2| + \frac{1}{5} \ln|x^2+1| + \frac{1}{5} \ln|x|$$



$$3 \int \frac{(2x^2+1)}{(x+3)(x-2)^2}$$

$$\int \frac{(2x^2+1)}{(x-3)(x-2)^2} = \frac{A}{(x-3)} + \frac{B}{(x+2)} + \frac{C}{(x-2)^2}$$

$$\frac{2x^2+1}{(x-3)(x-2)^2} = \frac{A(x+2)^2}{(x-3)(x-2)^2} + \frac{B(x-3)(x-2)}{(x-3)(x-2)^2} + \frac{C(x-3)}{(x-3)(x-2)^2}$$

$$x^2+1 = A(x+2)^2 + B(x-3)(x-2) + C(x-3)$$

$$x^2+1 = A(x^2+4x+4) + B(x^2-5x+6) + C(x-3)$$

$$x^2+1 = Ax^2 + Bx^2 + 4Ax - 5Bx + C + 4A + 6B - 3C$$

$$2x^2 \cdot x = 3$$

$$x^2+1 = A(x+2)^2$$

$$9+1 = A(3+2)^2$$

$$A = \frac{10}{25}$$

Compare LHS to RHS

$$x^2+1 = x^2(A+B) + x(4A-5B+C) + 4A+6B-3C$$

Compare LHS to RHS

$$(A+B) = 1 \dots (1)$$

$$4A - 5B + C = 0 \dots (2)$$

$$4A + 6B - 3C = 1 \dots (3)$$

Sub A into eq 1

$$\frac{10}{25} + B = 1$$

$$B = 1 - \frac{10}{25} = \frac{3}{5}$$

Sub A and B into eq 2

$$4 \times \frac{2}{5} - 5 \times \frac{3}{5} + C = 0$$

$$\frac{8}{5} - \frac{15}{5} + C = 0$$

$$C = \frac{7}{5}$$



$$\int \frac{(x^2+1)}{(x-3)(x-2)^2} dx = \frac{2}{5} \int \frac{dx}{(x-3)} + \frac{3}{5} \int \frac{dx}{x-2} + \frac{7}{5} \int \frac{dx}{(x-2)^2}$$

$$= \frac{2}{5} \ln(x-3) + \frac{3}{5} \ln(x-2) + \frac{7}{5} (x-2)^{-1} + C$$

$$\int \frac{x^3 + x^2 + x + 1}{(x-1)}$$

$$\begin{array}{r} x^2 + 2x + 3 \\ (x-1) \overline{) x^3 + x^2 + x + 1} \\ \underline{x^3 + x^2} \phantom{+ 1} \\ 2x^2 + x + 1 \\ \underline{-2x^2 + 2x} \phantom{+ 1} \\ 3x + 1 \\ \underline{-3x - 3} \\ +4 \end{array}$$

$$= \int x^2 dx + \int 2x dx + \int 3x + \int \frac{-4}{x-1} dx$$

$$= \frac{x^3}{3} + x^2 + 3x + 4 \ln(x-1)$$