Math 104

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$\int \frac{3x-1}{(x-1)(x-2)(x-3)}\,dx$

$\frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{A}{(x-1)} + \frac{B}{(x-2)} + \frac{C}{(x-3)}$

$\frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{A(x-2)(x-3)+B(x-1)(x-3)+C(x-1)(x-2)}{(x-1)(x-2)(x-3)}$

$A(x^2-5x+4)+B(x^2-4x+3)+C(x^2-3x+2)$

$3x-1 = Ax^2-5xA+6A+Bx^2-4xB+3B+Cx^2-3xC+2C$

$3x-1 = Ax^2+Bx^2+Cx^2-5xA-4xB-3xC+6A+3B+2C$

$3x-1 = x^2(A+B+C)+x(-5A-4B-3C)+6A+3B+2C$

$-1 = 6A+3B+2C \quad -(1)$

$3 = -5A-4B-3C \quad -(2)$

$0 = A+B+C \quad -(3)$

Eqn(3)

$A = -B-C \quad -(4)$

Substitute eqn(4) into (1) & (2).

$6(-B-C)+3B+2C = -1.$

$-5(-B-C)-4B-3C = 3$

$-6B-6C+3B+2C = -1$

$+5B+5C-4B-3C = 3$

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$-6B+3B-6C+2C = -1$

$+5B-4B+5C-3C = 3$

$-3B-4C = -1 \quad -(5)$

$B+2C = 3 \quad -(6)$

$B = 3-2C \quad -(7)$

Substitute eqn(7) into (5)

$-3(3-2C)-4C = -1$

$-9+6C-4C = -1$

$-9+2C = -1$

$2C = 8$

$C = 4$

$B = 3-2(4)$

$= 3-8$

$= -5$

$\therefore A = -(-5)-4$

$+5-4$

$= 1$

$\therefore A = 1 \quad B = -5 \quad C = 4$

In partial fractions

$\frac{3x-1}{(x-1)(x-2)(x-3)} dx = \frac{1}{x-1} + \frac{-5}{x-2} + \frac{4}{x-3}$

$\int \frac{3x-1}{(x-1)(x-2)(x-3)} dx = \int \frac{1}{x-1} dx - \int \frac{5}{x-2} dx + \int \frac{4}{x-3} dx$

$\ln(x-1) - 5\ln(x-2) + 4\ln(x-3)$

$\ln(x-1) - 5\ln(x-2) + 4\ln(x-3) + C$.

②

$\frac{x^2+x+1}{(x+2)(x^2+1)} dx$.

$\frac{x^2+x+1}{(x+2)(x^2+1)} = \frac{A}{(x+2)} + \frac{Bx+C}{(x^2+1)}$

$x^2+x+1 = \frac{A(x^2+1)+(Bx+C)(x+2)}{(x+2)(x^2+1)}$.

$x^2+x+1 = A(x^2+1)+(Bx+C)(x+2)$.

$= Ax^2+A+Bx^2+2Bx+Cx+2C$

$= Ax^2+Bx^2+2Bx+Cx+2C+A$.

$(A+B)x^2 + x(2B+C) + 2C+A$.

$A+2C$. — ①

$C+2B$ — ②

$A+B$ — ③ $\boxed{C=2B-1}$ — ④

$A+2C = 1$ — ⑤

$A = A+B = 1$.

Ubtitute equ(4) into 5 $A = 2/5$. $B = 3/5$ $C = 1/5$

$2(2B-1) = 1$.

$4B-2 = 1$.

$4B = 3$. $\Rightarrow$ $A+4B = 3$

$-A+B = 1$

$\frac{x^2+x^2+1}{(x+2)(x^2+1)} = \frac{2}{A} \quad \frac{3}{5(x+2)} + \frac{2}{5} + \frac{1}{5(x+1)}$

$\frac{3}{5(x+2)} + \frac{3(x+1)}{5(x^2+1)}$.

$\int \frac{3}{5(x+2)} dx + \int$

$\frac{3}{5}\ln(x+2) + \frac{1}{5}\ln(x^2+1) + \frac{2}{5}\tan^{-1}(x/5) + C$.

③

$\frac{x^2+1}{(x-3)(x-2)^2} dx$.

$\frac{x+1}{(x-3)(x-2)^2} = \frac{A}{(x-3)} + \frac{B}{(x-2)} + \frac{C}{(x-2)^2}$.

$x+1 = \frac{A(x-2)(x-2)^2 + B(x-3)(x-2)^2 + C(x-3)(x-2)}{(x-3)(x-2)(x-2)^2}$.

$\frac{x+1}{(x-3)(x-2)^2} = \frac{A}{(x-3)} + \frac{B}{(x-2)} + \frac{C}{(x-2)^2}$

$x+1 = \frac{A(x^2-4x+4)+B(x-3)(x-2)+C(x-3)}{(x-3)(x-2)^2}$.

$x+1 = A(x^2-4x+4)+B(x^2-5x+6)+C(x-3)$.

$x+1 = Ax^2-4xA+4A+Bx^2-5xB+6B+Cx-3C$.

$x+1 = Ax^2+Bx^2-4xA-5xB+Cx+4A+6B-3C$.

$x+1 = (A+B)x^2+x(-4A-5B+C)+4A+6B-3C$.

$A+B=1 \quad -(1)$

$-4A-5B+C=1 \quad -(2)$

$4A+6B-3C=0 \quad -(3)$

From equ (1):

$A=1-B \quad -(4)$ put (4) into 2 & (3)

$-4(1-B)-5B+C=1$

$+4(1-B)+6B-3C=0$

$-4+4B-5B+C=1$

$4-4B+6B-3C=0$

$B+C=1+4$

$2B-3C=0-4$

$B+C=5 \times 2 \quad = 2B+2C=10$

$2B-3C=-4 \times 1 \quad 2B-3C=-4$

$A=10 \quad B=-9 \quad C=-5$

$$\frac{x^2+1}{(x-3)(x-2)^2} = \frac{10}{x-3} \underbrace{- \frac{9}{(x-2)}} - \frac{5}{(x-2)^2}$$

$$10\ln(x-3)-9\ln(x-2)+\frac{5}{x-2}+C$$

$$\int \frac{2x^3 + 2x^2 + 2x + 1}{x - 1}$$

Separate into fractions

$$\int 2x^2 + \frac{2x^2}{2x-1} + \frac{2x}{2x-1} + \frac{1}{2x-1}\, dx$$

$$\int \frac{2x^3}{2x-1}\, dx + \int \frac{2x^2}{2x-1}\, dx = \int \left(\frac{2x(x+1)}{2x-1}\right) \frac{1}{2x-1}\, dx$$

$$\frac{2x^3}{3} + \frac{2x^2}{2} + 2x - \frac{1}{2}\ln|2x-1| + \frac{x^2}{2} + \frac{x}{2}$$

$$\frac{2x^3}{3} + \frac{3x^2}{2} + \frac{3x}{2} - \frac{1}{2}\ln|2x-1| + C$$

$$\frac{2x^3 + 3x^2 + 3x}{3} - \frac{1}{2}\ln|2x-1| + C$$