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MAT 102

① $M = Pi - 6j - 3k$

$N = 4i + 3j - k$

$O = i - 3j + 2k$

a. M and N are perpendicular to each other

$M.N = Pi - 6j - 3k$

$\underline{4i + 3j - k}$

$4P - 18 + 3$

$= 4P - 15$

Since they are perpendicular

$4P - 15 = 0$

$P = \frac{15}{4}$

b M, N and O are coplaner

$M.(N \times O)$

$$N \times O = \begin{vmatrix} i & j & k \\ 4 & 3 & -1 \\ 1 & -3 & 2 \end{vmatrix}$$

$$i\begin{vmatrix}3 & -1\\ -3 & 2\end{vmatrix} - j\begin{vmatrix}4 & -1\\ 1 & 2\end{vmatrix} + k\begin{vmatrix}4 & 3\\ 1 & -5\end{vmatrix}$$

$$i(6-3) - j(8+1) + k(-12-3)$$
$$= 3i - 9j - 15k$$

$$M.(N \times O) = \frac{3i - 9j - 15k}{Pi - 6j - 3k}$$
$$= 3Pi + 54j + 45k$$
$$\therefore \; 3P + 54 + 45 = 0$$
$$3P + 99 = 0$$
$$\frac{3P}{3} = \frac{-99}{3}$$
$$P = -33$$

2. $\bar{V} = (3i + 2j + 5k) + (i - j + 6k) + (5i + 2j - 3k)$
$$= 10i + 3j + 8k$$
$a_x = 10$, $a_y = 3$ and $a_z = 8$
$$|v| = \sqrt{10^2 + 3^2 + 8^2}$$
$$= \sqrt{100 + 9 + 64}$$
$$= \sqrt{173} = 13.15$$

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ii direction cosine are

$$\cos\alpha = \frac{ax}{|v|} = \frac{10}{13.15} = 0.761$$

$$\cos\beta = \frac{ay}{|v|} = \frac{3}{13.15} = 0.228$$

$$\cos\gamma = \frac{az}{|v|} = \frac{8}{13.15} = 0.608$$

ii unit vector

$$= \frac{v}{|v|} = \frac{10i + 3j + 8k}{13.15}$$

5. $f = 3ui + u^2j + (u+2)k$

$v = 2ui - 3uj + (u-2)vk$

$$f \times v = \begin{vmatrix} i & j & k \\ 3u & u^2 & (u+2) \\ 2u & -3u & (u-2)k \end{vmatrix}$$

$\cancel{f\times v = i(u^2(u-2) - (u+2)-3u}$

$f \times v = i(u^2(u-2) - [-3(u+2)] - j(3u(u-2) - (2u(u+2)$

$+k[(3u \times 3u) - (2u \times u^2]$

$$i\left[u^3-2u^2-(-3u-6u)\right]-j\left[3u^2-6u-2u^2-4u\right]+$$
$$k\left[-2u^3-9u^2\right]$$

$$=i\left[u^3-2u^2+3u^3+6u\right]-j\left[u^2-10u\right]+k\left[-2u^3-9u^2\right]$$
$$=i\left[u^3+u^2+6u\right]-j\left[u^2-10u\right]+k\left[-2u^3-9u^2\right]$$

$$\int_0^1 (f\times v)=\int i\left[u^3+u^2+6u\right]-\int j\left[u^2-10u\right]+\int k\left[-2u^3-9u^2\right]$$
$$=i\int u^3+u^2+6u-j\int u^2-10u+k\int -2u^3-9u^2$$

$$=\left[\frac{u^4}{4}+\frac{u^3}{3}+\frac{6u}{2}\right]-j\left[\frac{u^3}{3}-\frac{10u}{2}\right]+k\left[\frac{2u^4}{4}-\frac{9u^3}{3}\right]$$
$$=\left[\frac{u^4}{4}+\frac{u^3}{3}+3u^2\right]-j\left[\frac{u^3}{3}-5u^2\right]+k\left[-\frac{u^4}{2}-3u^3\right]+C$$
$$\int_0^1 (f\times v)=i\left[\frac{(1)^4}{4}+\frac{(1)^3}{3}+3(1)^2\right]-j\left[\frac{(1)^3}{3}-5(1)^2\right]+k\left[\frac{-(1)^4}{2}-3(1)^3\right]+C-[0+C]$$
$$\int_0^1 (f\times v)=i\left[\frac{1}{4}+\frac{1}{3}+3\right]-j\left[\frac{1}{3}-5\right]+k\left[\frac{-1}{2}-3\right]+C-C$$
$$\int_0^1 (f\times v)=i\left[\frac{43}{12}\right]-j\left[\frac{-14}{3}\right]+k\left[\frac{-7}{2}\right]$$

$$\int_0^1 (f\times v)=\frac{43}{12}i-\frac{14}{3}j-\frac{7}{2}k$$