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Mechatronics
19/ENG05/042
Maths 104

Course Code : Mat 104

Find the integral of the following

1. $\int \frac{3x-1}{(x-1)(x-2)(x-3)}dx = \frac{A}{(x-1)} + \frac{B}{(x-2)} + \frac{C}{x-3}$

$= \int \frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{A(x-2)(x-3)+B(x-1)(x-3)+C(x-1)(x-2)}{(x-1)(x-2)(x-3)}$

$\therefore 3x-1 = Ax^2-5Ax+6A+Bx^2-4Bx+3B+Cx^2-3xC+2C$

$3x-1 = x^2(A+B+C)+x(-5A-4B-3C)+6A+3B+2C$

$A+B+C = 0$ Matrix

$-5A-4B-3C = 3$ $D = \begin{vmatrix} 1 & 1 & 1 \\ -5 & -4 & -3 \\ 6 & 3 & 2 \end{vmatrix} = \begin{vmatrix} 0 \\ 3 \\ -1 \end{vmatrix}$

$6A+3B+2C = -1$

Using determinant and crammer's rule

normal determinant $= 1(-8+9)-1(-10+18)+1(-15+24) = 1-8+9 = 2$

To find A substitute RHS in A column

$D_a = \begin{pmatrix} 0 & 1 & 1 \\ 3 & -4 & -3 \\ -1 & 3 & 2 \end{pmatrix} = 0(-8+9)-1(6-3)+1(9-4) = 0-3+5 = 2$

$\therefore A = \frac{D_a}{D} = \frac{2}{2} = 1$

$D_b = \begin{pmatrix} 1 & 0 & 1 \\ -5 & 3 & -3 \\ 6 & -1 & 2 \end{pmatrix} = 1(6-3)-0(-10+18)+1(5-18) = 3-13 = -10 \therefore B = \frac{D_b}{D} = \frac{-10}{2} = -5$

$D_c = \begin{pmatrix} 1 & 1 & 0 \\ -5 & -4 & 3 \\ 6 & 3 & -1 \end{pmatrix} = 1(4-9)-1(5-18)+1(-15+18) = -5+13+3 = 11$

$c = \frac{D_c}{|D|} = \frac{8}{2} = 4$

$= \int \frac{1}{x-1} - \frac{5}{(x-2)} + \frac{4}{(x-3)}$

$= \ln(x-1) - 5\ln(x-2) + 4\ln(x-3) + C$

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2) $(x^2+x+1)/(x+2)(x^2+1)\,dx$

$$\int \frac{(x^2+x+1)}{(x+2)(x^2+1)}\,dx = \frac{A}{x+2} + \frac{Bx+C}{(x^2+1)}$$

$$= \int \frac{x^2+x+1}{(x+2)(x^2+1)} = \frac{A(x^2+1)+(Bx+C)(x+2)}{(x+2)(x^2+1)}$$

$$x^2+x+1 = Ax^2+A+Bx^2+2Bx+Cx+2C$$

$$= \text{at } x = -2$$

$$A(-2^2+1)+(B(-2)-2(-2+2)$$

$$(-2)^2-2+1 = +5A \therefore 3 = 5A \therefore A = \tfrac{3}{5}$$

$$A + x^2 - 1 = x = \sqrt{-1} = \frac{2x+1}{5}$$

$$B = 2,\ C = 1$$

$$= \int \frac{3}{5(x+2)} + \frac{2x+1}{5(x^2+1)}\ )\,dx$$

$$= \int \frac{3}{5(x+2)} + \frac{2x}{5(x^2+1)} + \frac{1}{5(x^2+1)}\,dx$$

$$= \frac{3\ln(x+2)}{5} + \frac{\ln(x^2+1)}{5} + \frac{\arctan x}{5} + c$$

3) $$\int \frac{x^2+1}{(x-3)(x-2)^2} = \frac{A}{x-3} + \frac{B}{(x-2)} + \frac{C}{(x-2)^2}$$

$$x^2+1 = \frac{A(x-2)(x-2)^2 + B(x-3)(x-2)^2 + C}{(x-3)(x-2)}$$

$$A = 10,\ B = -9,\ C = -5$$

$$= \int \frac{10}{x-3} - \frac{9}{x-2} - \frac{5}{(x-2)^2}$$

$$= 10\ln(x-3) - 9\ln(x-2) + \frac{5}{x-2} + C$$

4) $$\int \frac{(x^3+x^2+x+1)}{x-1}\,dx$$

~~Substitute~~ $u = x-1 \therefore du = dx$

~~use~~ $x^2 = (u+1)^2$
$x^3 = (u+1)^3$

$$= \int \frac{u^3}{4} + 4u$$

Perform Polynomial long division

$$\int = \left(x^2+2x+\frac{4}{x-1}+3\right)dx$$

$$= \frac{x^3}{3} + \frac{2x^2}{2} + 4\ln(x-1) + 3x + c$$

$$= \frac{x^3}{3} + x^2 + 4\ln(x-1) + 3x + c.$$