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Find the integral of the following:

1. $\int \frac{(3x-1)}{(x-1)(x-2)(x-3)} dx$

solution.

$$\frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$$

$$3x-1 = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)$$

when $x=2$ | when $x=1$

$3(2)-1 = B(1)(-1)$ | $3(1)-1 = A(-1)(-2)$

$5 = -B$ | $2 = 2A$

$B = -5$ | $A = 1$

when $x=3$

$3(3)-1 = C(2)(1)$

$8 = 2C$

$C = 4$ $\therefore A=1,\ B=-5,\ C=4$

$$\therefore \frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{1}{x-1} + \frac{(-5)}{x-2} + \frac{4}{x-3}$$

$$\int \frac{3x-1}{(x-1)(x-2)(x-3)} dx = \int \frac{1}{x-1} dx + 4\int \frac{1}{x-3} dx - 5\int \frac{1}{x-2} dx$$

$$\int \frac{3x-1}{(x-1)(x-2)(x-3)} dx = \ln(x-1) + 4\ln(x-3) - 5\ln(x-2) + C$$

2. $\int \frac{(x^2+x+1)}{(x+2)(x^2+1)}\,dx.$ $\quad \frac{x^2+x+1}{(x+2)(x^2+1)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+1}$

$x^2+x+1 = A(x^2+1) + (Bx+C)(x+2)$

when $x=-2$

$4-2+1 = A(5) + (-2B+C)(0)$

$3 = 5A$

$A = \frac{3}{5}$

when $x=-1$

$1 \cancel{+1} +1 = A[\cancel{0}] + (-B+C)(1)$

$1 = \cancel{1}A - B + C$ 0

$\cancel{2}A - B + C = 1$

$\therefore C = 1+B$

when $x=1$

$1+1+1 = \frac{3}{5}(2) + [B(1) + (1+B)(3)]$

$3 = \frac{6}{5} + \cancel{3B} + 3 + 3B$

$3 = \frac{6}{5} + 6B + 3$

$0 = \frac{6}{5} + 6B$

$B = -\frac{1}{5}$

$C = 1+B$

$C = 1 - \frac{1}{5}$

$C = \frac{4}{5}$; $A = \frac{3}{5}, B = -\frac{1}{5}, C = \frac{4}{5}$

$\therefore \int \frac{x^2+x+1}{(x+2)(x^2+1)}\,dx$

$= \frac{3}{5}\left(\frac{1}{x+2}\right) + \left[\frac{-\frac{1}{5}(x) + \frac{4}{5}}{x^2+1}\right]$

$= \frac{3}{5(x+2)} + \left(\frac{-x+4}{5(x^2+1)}\right)$

$\frac{x^2+x+1}{(x+2)(x^2+1)}\,dx = \frac{3}{5(x+2)} + \frac{4-x}{5(x^2+1)}$

$\int \frac{x^2+x+1}{(x+2)(x^2+1)}\,dx = \frac{3}{5}\int \frac{1}{x+2}\,dx + \frac{4-x}{5}\int \frac{1}{x^2+1}$

$\therefore \int \frac{x^2+x+1}{(x+2)(x^2+1)} = \frac{3}{5}\ln(x+2) + \frac{4-x}{5}\,\frac{1}{1}\tan^{-1}\left(\frac{x}{1}\right)$

$\therefore \frac{3}{5}\ln(x+2) + \frac{4-x}{5}\tan^{-1}(x) + C$

3. $\int \frac{x^2+1}{(x-3)(x-2)^2}dx$ $\quad \frac{x^2+1}{(x-3)(x-2)^2} = \frac{A}{x-3} + \frac{B}{(x-2)} + \frac{C}{(x-2)^2}$

$x^2+1 = A(x-2)^2 + B(x-3)(x-2) + C(x-3)$

when $x=3$

$10 = A(1) + B(0)(1) + C(0)$

$10 = A \therefore A = 10$

when $x=2$

$5 = 0 + B(-1)(0) + C(-1)$

$5 = -C \therefore C = -5$

when $x=1$

$2 = 10(1) + B(-2)(-1) + (-5(-2))$

$2 = 10 + 2B + 10$

$2 = 20 + 2B$

$-18 = 2B$

$B = -9 \quad \therefore A = 10,\ B = -9,\ C = -5$

$\therefore \int \frac{x^2+1}{(x-3)(x-2)^2}dx = \frac{10}{x-3} + \frac{(-9)}{x-2}dx + \frac{(-5)}{(x-2)^2}dx$

let $u = x+2$

$\frac{du}{dx} = 1 \quad \therefore du = dx$

$= 10\ln(x+3) - 9\ln(x+2) - 5\int \frac{1}{(x+2)^2}dx$

$\therefore \int \frac{1}{(x+2)^2} = \int \frac{1}{u^2}du = \int u^{-2}du$

$= \left[\frac{u^{-2+1}}{-1} + c\right]$

$= -u^{-1} + c$

$\int \frac{1}{(x+2)^2}dx = -(x+2)^{-1} + c$

$\int \frac{x^2+1}{(x-3)(x-2)^2} \cancel{= 10\ln(x+3) - 9\ln(x+2)}$

$= 10\ln(x+3) - 9\ln(x+2) - 5\left[-(x+2)^{-1}\right] + c$

$\therefore \int \frac{x^2+1}{(x-3)(x-2)^2}dx =$

$10\ln(x+3) - 9\ln(x+2) + \frac{5}{x+2} + c$

4. $\int \frac{x^3+x^2+x+1}{x-1}dx$

$$\begin{array}{r|l} & x^2+2x+3 \\ \hline x-1 & x^3+x^2+x+1 \\ & \underline{x^3-x^2} \\ & \quad 2x^2+1 \\ & -\ \underline{2x^2-2x} \\ & \qquad 3x+1 \\ & \qquad \underline{-3x-3} \\ & \qquad\quad 4 \end{array}$$

$\int \frac{x^3+x^2+x+1}{x-1}dx = \int x^2+2x+3 + \frac{4}{x-1}$

$= \int x^2dx + \int 2x\,dx + \int 3dx + 4\int \frac{1}{x-1}dx$

$= \frac{x^3}{x} + \frac{2x^2}{2} + 3x + 4\ln(x-1) + c$

$= \frac{x^3}{3} + x^2 + 3x + 4\ln(x-1) + c$