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Computer science

Find the integral of the following

1) $\int \frac{3x-1}{(x-1)(x-2)(x-3)} dx$

$\frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$

$\frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)}{(x-1)(x-2)(x-3)}$

$3x-1 = A(x^2-5x+6) + B(x^2-4x+3) + C(x^2-3x+2)$

$3x-1 = Ax^2-5Ax+6A + Bx^2-4Bx+3B + Cx^2-3Cx+2C$

collect like terms

$3x-1 = x^2(A+B+C) + x(-5A-4B-3C) + 6A+3B+2C$

Comparing the LHS with the RHS

$A+B+C=0$ — (i)

$-5A-4B-3C=3$ — (ii)

$6A+3B+2C=-1$ — (iii)

$A=-B-C$ — (iv)

Put (iv) in (ii) and (iii)

$-5(-B-C)-4B-3C = B+2C = 3$ — (v)

$6(-B-C)+3B+2C = -3B-4C = -1$ — (vi)

from (v)

$B = 3-2C$

Substituting $B=3-2C$ in (vi)

$-3(3-2C) - 4C = -1$

$-9+6C-4C = -1$

$-9+2C = 1$

$C=4$

Substituting $C=4$ in equation (v)

$B + 2(4) = 3$

$B + 8 = 3$

$B = 3 - 8 = -5$

Substituting $B = -5$ and $C = 4$ in (1)

$A - 5 + 4 = 0$

$A = 5 - 4 = 1$

Hence

$$\int \frac{3x-1}{(x-1)(x-2)(x-3)} dx = \int \frac{1}{x-1} dx - \int \frac{5}{x-2} dx + \int \frac{4}{x-3} dx$$

$$= \ln|x-1| - 5\ln|x-2| + 4\ln|x-3| + C$$

2 $$\int \frac{x^2+x+1}{(x+2)(x^2+1)} dx$$

solution

$$\frac{x^2+x+1}{(x+2)(x^2+1)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+1}$$

$$\frac{x^2+x+1}{(x+2)(x^2+1)} = \frac{A(x^2+1) + Bx+C(x+2)}{(x+2)(x^2+1)}$$

$$x^2+x+1 = A(x^2+1) + Bx+C(x+2)$$

$$x^2+x+1 = Ax^2+A + Bx^2 + 2Bx + Cx + 2C$$

collecting like terms

$$x^2+x+1 = x^2(A+B) + x(2B+C) + A + 2C$$

Comparing the L.H.S and R.H.S

$A + B = 1$ — (i)

$2B + C = 1$ — (ii)

$A + 2C = 1$ — (iii)

$B = 1 - A$ — (iv)

Substituting (iv) into (ii)

$2(1-A) + C = -2A + C = -1$ (v)

from (iii)

$A = 1 - 2C$

Substituting $A = 1-2C$ in (v)

$-2(1-2C) + C = -2 + 4C + \cancel{C} = 1$

$5C = 1$

$C = 1/5$

Substituting $C = 1/5$ in iii

$A + 2(1/5) = 1$

$A = 3/5$

Substituting $A = 3/5$ in (i)

$3/5 + B = 1$

$B = 2/5$

Hence

$$\int \frac{x^2+x+1}{(x+2)(x^2+1)}\,dx = \int \frac{-3/5}{x+2}\,dx + \int \frac{\frac{2}{5}x + \frac{1}{5}}{x^2+1}\,dx$$

$$= \int \frac{-3/5}{x+2}\,dx + \int \frac{2x+1}{5(x^2+1)}\,dx$$

$$= \int \frac{3}{5(x+2)}\,dx + \int \frac{2x+1}{5(x^2+1)}\,dx$$

$$= 3/5 \ln|x+2| + 1/5 \ln|x^2+1| + 1/5 \tan^{-1} x + c$$

3 $$\int \frac{x^2+1}{(x-3)(x-2)^2}\,dx$$

$$\frac{x^2+1}{(x-3)(x-2)^2} = \frac{A}{x-3} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$$

$$\frac{x^2+1}{(x-3)(x-2)^2} = \frac{A(x-2)^2 + B(x-3)(x-2) + C(x-3)}{(x-3)(x-2)(x-2)^2}$$

$x^2+1 = A(x-2)^2 + B(x-3)(x-2) + C(x-3)$

$x^2+1 = A(x^2-4x+4) + B(x^2-5x+6) + C(x-3)$

$x^2+1 = Ax^2 - 4Ax + 4A + Bx^2 - 5Bx + 6B + Cx - 3C$

collecting like terms

$x^2+1 = x^2(A+B) + x(-4A-5B+C) + 4A + 6B - 3C$

Comparing LHS and RHS

$A + B = 1 - (i)$

$-4A - 5B + C = 0 \text{ — (ii)}$

$4A + 6B - 3C = 1 \text{ — (iii)}$

$A = 1 - B \text{ — (iv)}$

Substituting (iv) in (ii) and (iii)

$-4(1-B) - 5B + C = -B + C = 4$ — (v)

$4(1-B) + 6B - 3C = 2B - 3C = -3$ — (vi)

from (v)

$B = -4 + C$

Substituting $B = -4+C$ in (vi)

$2(-4+C) - 3C = -3$

$-8 + 2C - 3C = -3$

$C = -5$

Substituting $C = -5$ in (v)

$-B - 5 = 4$

$B = -9$

Substituting $B = -9$ in (i)

$A - 9 = 1$

$A = 10$

Hence

$$\int \frac{x^2+1}{(x-3)(x-2)^2}\,dx = \int \frac{10\,dx}{x-3} - \int \frac{9\,dx}{x-2} - \int \frac{5\,dx}{(x-2)^2}$$

$$= 10\ln(x-3) - 9\ln(x-2) - \frac{5}{x-2} + C$$

4 $\displaystyle\int \frac{x^3+x^2+x+1}{x-1}\,dx$

$$\begin{array}{r}
x^2+2x+3 \\
x-1\,\overline{\big)\,x^3+x^2+x+1} \\
-x^3-x^2 \\
\hline
2x^2+x+1 \\
-2x^2-2x \\
\hline
3x+1 \\
-3x-3 \\
\hline
4
\end{array}$$

$$\int \frac{x^2+x^2+x+1}{x-1} = \int x^2\,dx + \int 2x\,dx + \int 3\,dx + \int \frac{4}{x-1}\,dx$$

$$= \frac{x^3}{3} + \frac{\cancel{2}x^2}{\cancel{2}} + 3x + 4\ln|x-1| + c$$

$$= \frac{x^3}{3} + x^2 + 3x + 4\ln|x-1| + c$$