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Assignment answers

1.) $\int \frac{3x-1}{(x-1)(x-2)(x-3)} dx$

Soln

$$\frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$$

$$3x-1 = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)$$

$$x=1$$

$$2 = A(-1)(-2)$$

$$\therefore A = -1 //$$

$$x=2$$

$$5 = B(-1)$$

$$B = -5 //$$

$$x=3$$

$$8 = 2C$$

$$\therefore C = 4 //$$

∴

$$\therefore \int \frac{3x-1}{(x-1)(x-2)(x-3)} dx = \int \left(\frac{-1}{x-1} + \frac{-5}{x-2} + \frac{4}{x-3} \right) dx$$

$$= -1 \ln(x-1) - 5 \ln(x-2) + 4 \ln(x-3) + C //$$

2.) $\int \frac{(x^2+x+1)}{(x+2)(x^2+1)} dx$

Soln

$$\frac{x^2+x+1}{(x+2)(x^2+1)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+1}$$

$$x^2 + x + 1 = A(x^2 + 1) + (Bx + C)(x + 2)$$

$$x = -2$$

$$4 - 2 + 1 = 5A$$

$$3 = 5A$$

$$A = \frac{3}{5}$$

$$\therefore x^2 + x + 1 = \frac{3(x^2 + 1)}{5} + (Bx + C)(x + 2)$$

$$x^2 + x + 1 = \frac{3x^2}{5} + \frac{3}{5} + Bx^2 + 2Bx + Cx + 2C$$

$$x^2 + x + 1 = \left(\frac{3}{5}\right)x^2 + \frac{3}{5} + (B)x^2 + (2B)x + (C)x + 2C$$

By comparison

$$x^2: 1 = \frac{3}{5} + B \quad \text{--- (1)}$$

$$x: 1 = 2B + C \quad \text{--- (2)}$$

$$\text{Constant: } 1 = \frac{3}{5} + 2C \quad \text{--- (3)}$$

Eqn (1) $\times 2$

$$2 = \frac{6}{5} + 2B \quad \text{--- (4)}$$

Eqn (4) - Eqn (2)

$$1 = \frac{6}{5} - C$$

$$C = \frac{6}{5} - 1 = \frac{1}{5}$$

Eqn (2) $\times 2$

$$2 = 4B + 2C \quad \text{--- (5)}$$

Eqn (5) - Eqn (3)

$$1 = 4B - \frac{3}{5}$$

$$1 + \frac{3}{5} = 4B$$

$$\frac{8}{5} = 4B$$

$$\therefore B = \frac{2}{5}$$

$$\int \frac{x^2+x+1}{(x+2)(x^2+1)} dx = \int \frac{\frac{3}{5}}{(x+2)} dx + \int \frac{2x+1}{5(x^2+1)} dx$$

$$= \frac{3}{5} \ln(x+2) + \frac{1}{5} \tan^{-1} x + c //$$

$$3) \int \frac{(x^2+1)}{(x-3)(x-2)^2} dx$$

Soln

$$\frac{x^2+1}{(x-3)(x-2)^2} = \frac{A}{(x-3)} + \frac{B_1}{(x-2)} + \frac{B_2}{(x-2)^2}$$

$$x^2+1 = A_1(x-2)^2 + A_2(x-3)(x-2) + A_3(x-3)$$

$$x=2$$

$$4+1 = -1A_3$$

$$A_3 = -5$$

$$x=3$$

$$10 = A_1 //$$

$$x^2+1 = 10(x-2)^2 + A_2(x-3)(x-2) - 5(x-3)$$

$$x^2+1 = 10x^2 - 40x + 40 + A_2x^2 - A_25x + A_26 - 5x + 15$$

By comparison

$$x^2: 1 = 10 + A_2$$

$$\therefore A_2 = 1 - 10$$

$$= -9 //$$

$$\int \frac{x^2+1}{(x-3)(x-2)^2} dx = \int \left(\frac{10}{(x-3)} - \frac{9}{(x-2)} - \frac{5}{(x-2)^2} \right) dx$$

$$= 10 \ln(x-3) - 9 \ln(x-2) + 5 \ln(x-2) + c //$$

$$(4) \int \frac{x^3 + x^2 + x + 1}{(x-1)}$$

$$(x-1) \overline{) \begin{array}{r} x^2 + 2x + 3 \\ x^3 + x^2 + x + 1 \end{array}}$$

$$\ominus \frac{x^3 - x^2}{}$$

$$0 + 2x^2 + x + 1$$

$$\ominus \frac{2x^2 - 2x}{}$$

$$0 + 3x + 1$$

$$\ominus \frac{3x - 3}{}$$

$$\underline{\underline{4}}$$

$$\int (x^2 + 2x + 3 + 4/(x-1)) dx$$

$$= \frac{x^3}{3} + x^2 + 3x + 4 \ln(x-1) + C //$$