Zl-Yahub Lateefah

19/eng05/027

Mechatronics Engineering

Find the integral of the following

1) $\int \frac{(3x-1)}{(x-1)(x-2)(x-3)} dx$; $\frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$

$\therefore 3x-1 = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)$

when $x=2$

$3(2)-1 = B(1)(-1)$

$5 = -B$

$B = -5$

where $n=1$

$3(1)-1 = A(-1)(-2)$

$2 = 2A$

$A = 2/2 = 1$

when $x=3$

$3(3)-1 = C(2)(1)$

$8 = 2C$

$C = 8/2 = 4$

$\therefore \frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{1}{x-1} + \left(\frac{-5}{x-2}\right) + \frac{4}{x-3}$

$\therefore \int \frac{3x-1}{(x-1)(x-2)(x-3)} = \ln(x-1) + 4\ln(x-3) - 5\ln(x-2) + C$

2) $\int \frac{(x^2+x+1)}{(x+2)(x^2+1)} dx$; $\frac{x^2+x+1}{(x+2)(x^2+1)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+1}$

$x^2+x+1 = A(x^2+1) + [(Bx+C)(x+2)]$

when $x=-2$

$(-2)^2 + (-2) + 1 = A[(-2)^2+1] + B(-2) + C(-2+2)$

$-34-2+1 = 5A$

$\therefore 3 = 5A$

$\therefore A = \frac{3}{5}$

when $x=-1$

$(-1)^2+(-1)+1 = A[(-1)^2+1]+[B(-1)+C](-1+2)$

$1 = -B+C$

$\therefore C = 1+B \quad \cdots (3)$

when $x=1$

$\therefore 1^2+1+1 = \frac{3}{5}(1^2+1)+[B(1)+(1+B)(1+2)]$

$3 = 3/5(2)+[(B+1+B)(3)]$

$3 = 6/5+6B+3$

$0 = \frac{6}{5}+6B$

$-6B/-6 = 6/5 \neq -6$

$\therefore B = 6/5 \times 1/6 = 1/5$

$\therefore B = -1/5$

$C = 1+B$

$\therefore C = 1+(-1/5)$

$= 1/1 - 1/5$

$C = \frac{5-1}{5} = \frac{4}{5}$

$$\therefore \int \frac{x^2+x^2+1}{(x+2)(x^2+1)}\,dx = \frac{3}{5}\left(\frac{1}{x+2}\right)+\left[\frac{-1/5(x)+4/5}{x^2+1}\right]$$

$$= \frac{3}{5(x+2)}+\left[\frac{-x/5+4/5}{x^2+1}\right]$$

$$= \frac{3}{5(x+2)}+\left[\frac{\frac{-x+4}{5}}{x^2+1}\right]$$

$$\frac{x^2+x+1}{(x+2)(x^2+1)}\,dx = \frac{3}{5(x+2)}+\frac{4-x}{5(x^2+1)}$$

$$\int \frac{x^2+x+1}{(x+2)(x^2+1)}\,dx = 3/5\int\frac{1}{x+2}+\frac{4-x}{5}\int\frac{1}{x^2+1}$$

$$\therefore \int \frac{x^2+x+1}{(x+2)(x^2+1)}\,dx = 3/5\ln(x+2)+\frac{4-x}{5}\left[1/1\tan^{-1}(x/1)\right]$$

$$\int \frac{x^2+x+1}{(x+2)(x^2+1)}\,dx = \frac{3}{5}\ln(x+2)+\frac{4-x}{5}\tan^{-1}(x)+C_4$$

3) $$\int \frac{x^2+1}{(x-3)(x-2)^2}\,dx, \quad \frac{x^2+1}{(x-3)(x-2)^2} = \frac{A}{x-3}+\frac{B}{x+2}+\frac{C}{(x-2)^2}$$

$\therefore x^2+1 = A(x-2)^2+B(x-3)(x-2)+C(x-3)$

when $x=3$

$3^2+1 = A(3-2)^2+B(3-3)(3-2)+C(3-3)$

$10 = A(1)^2$

$A = 10$

when $x=2$

$2^2+1 = A(2-2)^2+B(2-3)(2-2)+C(2-3)$

$5 = -C$

$\therefore C = -5$

when $x=1$

$1^2+1 = 10(1-2)^2 + B(1-3)(1-2) + [-5(1-3)]$

$2 = 10(-1)^2 + B(-2)(-1) + [-5(-2)]$

$2 = 10 + 2B + 10$

$2 = 20 + 2B$

$2B = 2 - 20$

$\frac{2B}{2} = \frac{-18}{2}$

$\therefore B = -9$

$\therefore \int \frac{x^2+1}{(x-3)(x-2)^2} dx = \int \left[\frac{10}{x+3} + \left(\frac{-9}{x+2}\right) + \left(\frac{-5}{(x+2)^2}\right)\right] dx$

let $u = x+2$

$du \Delta x = 1$

$du = dx$

$= 10\ln(x+3) - 9\ln(x+2) - 5\int \frac{1}{(x+2)^2} dx$

$\therefore \int \frac{1}{(x+2)^2} = \int \frac{1}{u^2} du = \int u^{-2} du$

$= [u^{-2+1}/-1 + C] = -u^{-1} + C$

$\therefore \int \frac{1}{(x+2)^2} du = -(x+2)^{-1} + C$

$\therefore \int \frac{x^2+1}{(x-3)(x-2)^2} dx = 10\ln(x+3) - 9\ln(x+2) - 5(-(x+2)^{-1}) + C$

$\int \frac{x^2+1}{(x-3)(x-2)^2} dx = 10\ln(x+3) - 9\ln(x+2) + \frac{5}{x+2} + C_4$

$\int \frac{x^3+x^2+x+1}{x-1} dx$

$$\begin{array}{r} x^2+2x+3 \\ x-1 \overline{)\, x^3+x^2+x+1} \\ x^3-x^2 \\ \hline 2x^2+x \\ 2x^2-2x \\ \hline 3x+1 \\ 3x-3 \\ \hline 4 \end{array}$$

$\therefore \int \frac{x^3+x^2+x+1}{x-1} dx = \int x^2+2x+3+\frac{4}{x-1}$

$= \int x^2 dx + \int 2x\,dx + \int 3\,dx + 4\int \frac{1}{x-1} dx$

$= x^3/3 + 2x^2/2 + 3x + 4\ln(x-1) + C_4$

$\int \frac{x^3+x^2+x+1}{x-1} dx = x^3/3 + x^2 + 3x + 4\ln(x-1) + C_4$