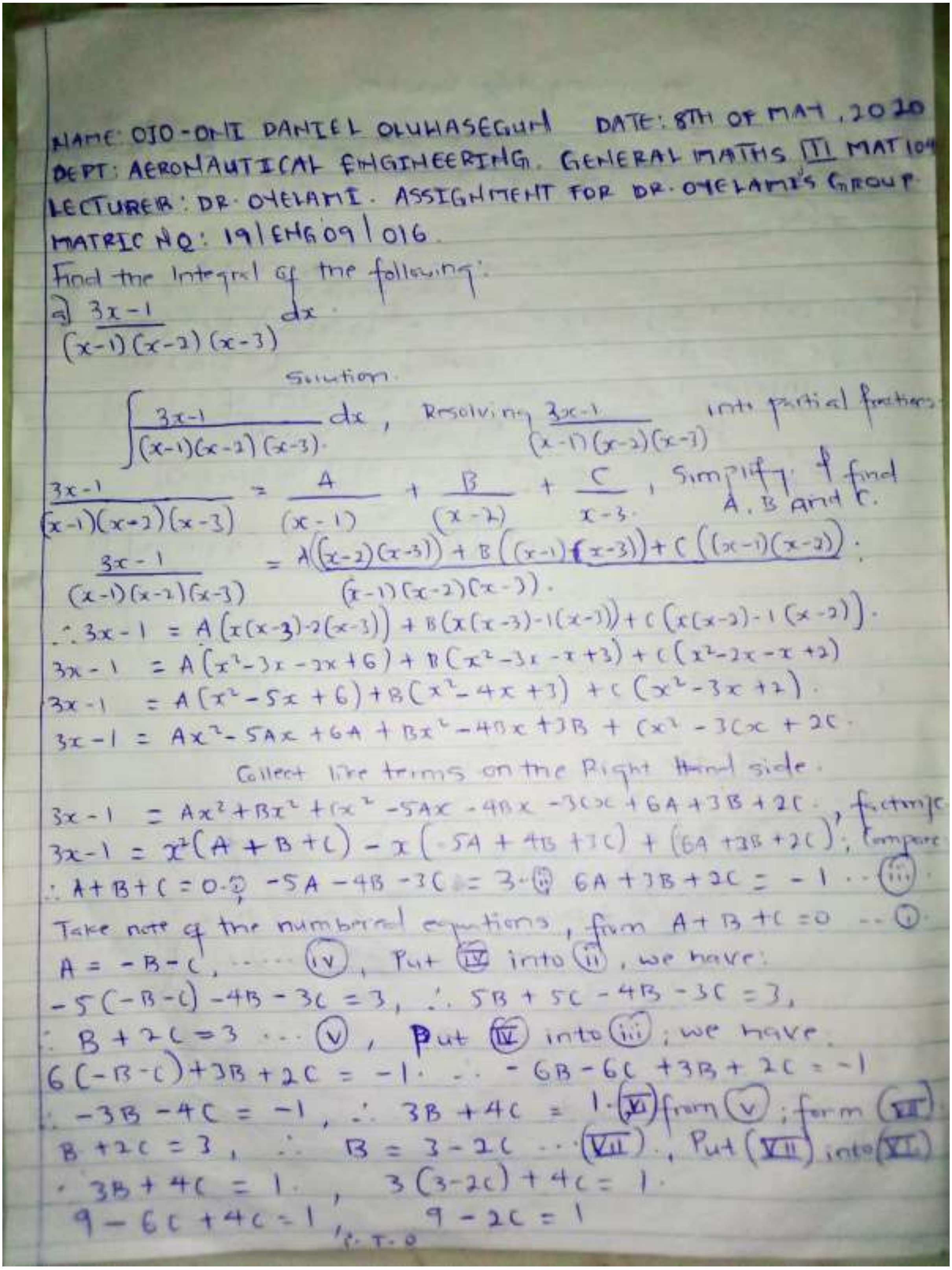
NAME: OJO-ONI DANIEL OLUWASEGUN, MATRIC NO.; 19/ENG09/016, DEPARTMMENT; AERONAUTICAL ENGINEERING, COURSE, MAT 104 GENERAL MATHEMATICS III, LECTURER; DR. OYELAMI, ASSIGNMENT FOR DR. OYELAMI'S GROUP, DATE SUBMITTED; 9TH OF MAY, 2020.



NAME: OJO-ONI DANIEL OLUWASEGUN DATE: 8TH OF MAY, 2020
DEPT: AERONAUTICAL ENGINEERING. GENERAL MATHS III MAT 104
LECTURER: DR. OYELAMI. ASSIGNMENT FOR DR. OYELAMI'S GROUP.
MATRIC NO: 19/ENG09/016.

Find the Integral of the following:

a) $\frac{3x-1}{(x-1)(x-2)(x-3)} dx$.

Solution.

$\int \frac{3x-1}{(x-1)(x-2)(x-3)} dx$, Resolving $\frac{3x-1}{(x-1)(x-2)(x-3)}$ into partial fractions.

$\frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{A}{(x-1)} + \frac{B}{(x-2)} + \frac{C}{x-3}$, Simplify & find A, B and C.

$\frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{A((x-2)(x-3)) + B((x-1)(x-3)) + C((x-1)(x-2))}{(x-1)(x-2)(x-3)}$.

$\therefore 3x-1 = A(x(x-3)-2(x-3)) + B(x(x-3)-1(x-3)) + C(x(x-2)-1(x-2))$.

$3x-1 = A(x^2-3x-2x+6) + B(x^2-3x-x+3) + C(x^2-2x-x+2)$

$3x-1 = A(x^2-5x+6) + B(x^2-4x+3) + C(x^2-3x+2)$.

$3x-1 = Ax^2-5Ax+6A+Bx^2-4Bx+3B+Cx^2-3Cx+2C$.

Collect like terms on the Right Hand side.

$3x-1 = Ax^2+Bx^2+Cx^2-5Ax-4Bx-3Cx+6A+3B+2C$, factorise

$3x-1 = x^2(A+B+C) - x(-5A+4B+3C) + (6A+3B+2C)$, compare

$\therefore A+B+C=0$ ...(i), $-5A-4B-3C=3$ ...(ii) $6A+3B+2C=-1$ ...(iii).

Take note of the numbered equations, from $A+B+C=0$ ...(i).

$A=-B-C$, ...(iv), Put (IV) into (ii), we have:

$-5(-B-C)-4B-3C=3$, $\therefore 5B+5C-4B-3C=3$,

$\therefore B+2C=3$ ...(v), Put (IV) into (iii); we have.

$6(-B-C)+3B+2C=-1$. $\therefore -6B-6C+3B+2C=-1$

$\therefore -3B-4C=-1$, $\therefore 3B+4C=1$.(VI) from (v); form (VII)

$B+2C=3$, $\therefore B=3-2C$ ...(VII), Put (VII) into (VI)

$\therefore 3B+4C=1$, $3(3-2C)+4C=1$.

$9-6C+4C=1$, $9-2C=1$

P.T.O

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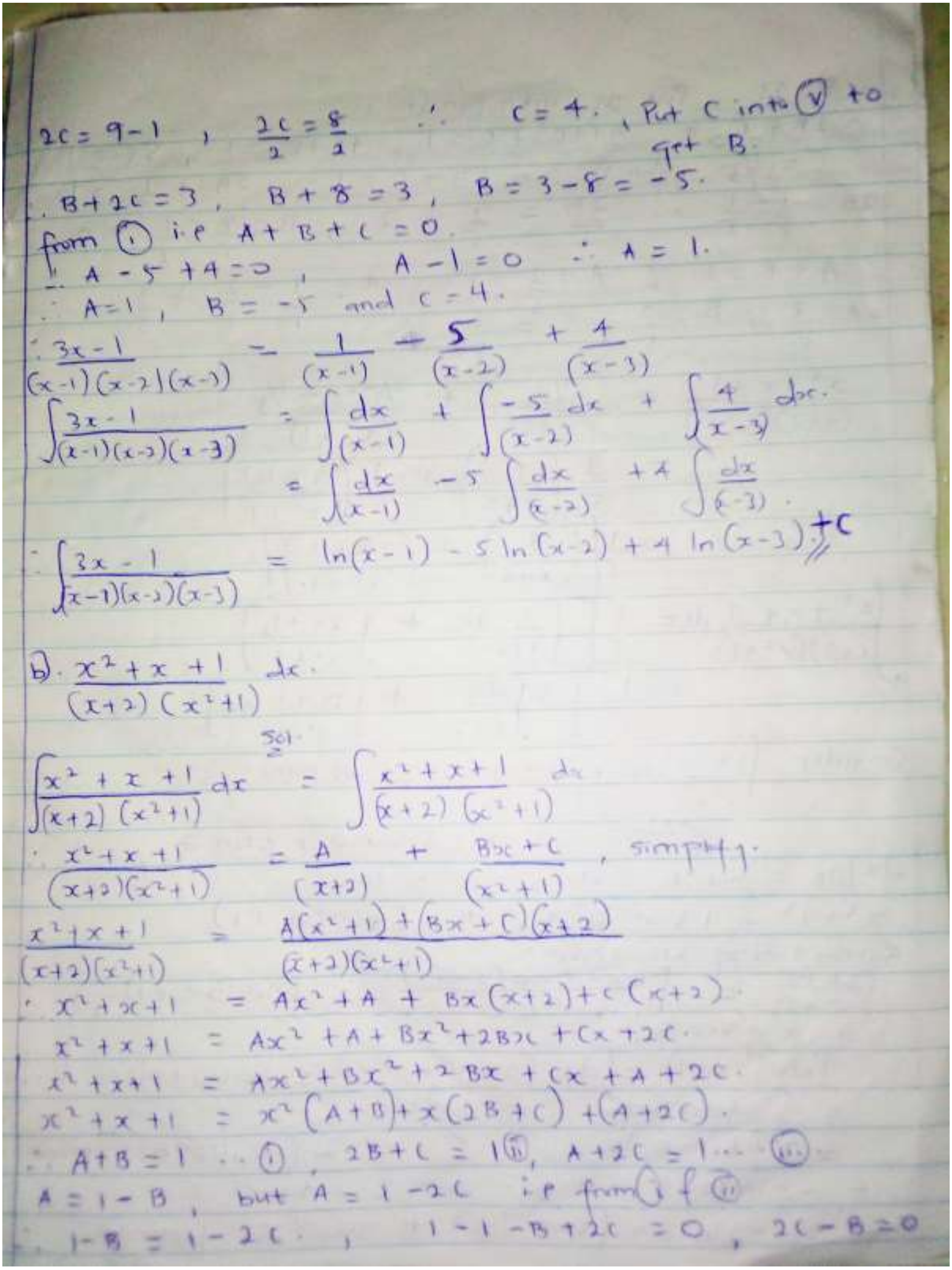

$2C = 9-1$, $\frac{2C}{2} = \frac{8}{2}$ $\therefore$ $C = 4$. , Put C into (v) to get B.

$\therefore B + 2C = 3$, $B + 8 = 3$, $B = 3 - 8 = -5$.

from ① i.e $A + B + C = 0$.

$\therefore A - 5 + 4 = 0$, $A - 1 = 0$ $\therefore A = 1$.

$\therefore A = 1$, $B = -5$ and $C = 4$.

$$\therefore \frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{1}{(x-1)} - \frac{5}{(x-2)} + \frac{4}{(x-3)}$$

$$\int \frac{3x-1}{(x-1)(x-2)(x-3)} = \int \frac{dx}{(x-1)} + \int \frac{-5}{(x-2)} dx + \int \frac{4}{x-3} dx.$$

$$= \int \frac{dx}{(x-1)} - 5 \int \frac{dx}{(x-2)} + 4 \int \frac{dx}{(x-3)}.$$

$$\therefore \int \frac{3x-1}{(x-1)(x-2)(x-3)} = \ln(x-1) - 5\ln(x-2) + 4\ln(x-3) + C$$

b). $\frac{x^2+x+1}{(x+2)(x^2+1)} dx$.

Sol:

$$\int \frac{x^2+x+1}{(x+2)(x^2+1)} dx = \int \frac{x^2+x+1}{(x+2)(x^2+1)} dx$$

$$\therefore \frac{x^2+x+1}{(x+2)(x^2+1)} = \frac{A}{(x+2)} + \frac{Bx+C}{(x^2+1)}, \text{ simplify.}$$

$$\frac{x^2+x+1}{(x+2)(x^2+1)} = \frac{A(x^2+1)+(Bx+C)(x+2)}{(x+2)(x^2+1)}$$

$$\therefore x^2+x+1 = Ax^2 + A + Bx(x+2) + C(x+2).$$

$$x^2+x+1 = Ax^2 + A + Bx^2 + 2Bx + Cx + 2C.$$

$$x^2+x+1 = Ax^2 + Bx^2 + 2Bx + Cx + A + 2C.$$

$$x^2+x+1 = x^2(A+B) + x(2B+C) + (A+2C).$$

$\therefore A + B = 1$ ...①, $2B + C = 1$ (ii), $A + 2C = 1$ ... (iii)

$A = 1 - B$, but $A = 1 - 2C$ i.e from (i) & (iii)

$\therefore 1 - B = 1 - 2C$, $1 - 1 - B + 2C = 0$, $2C - B = 0$

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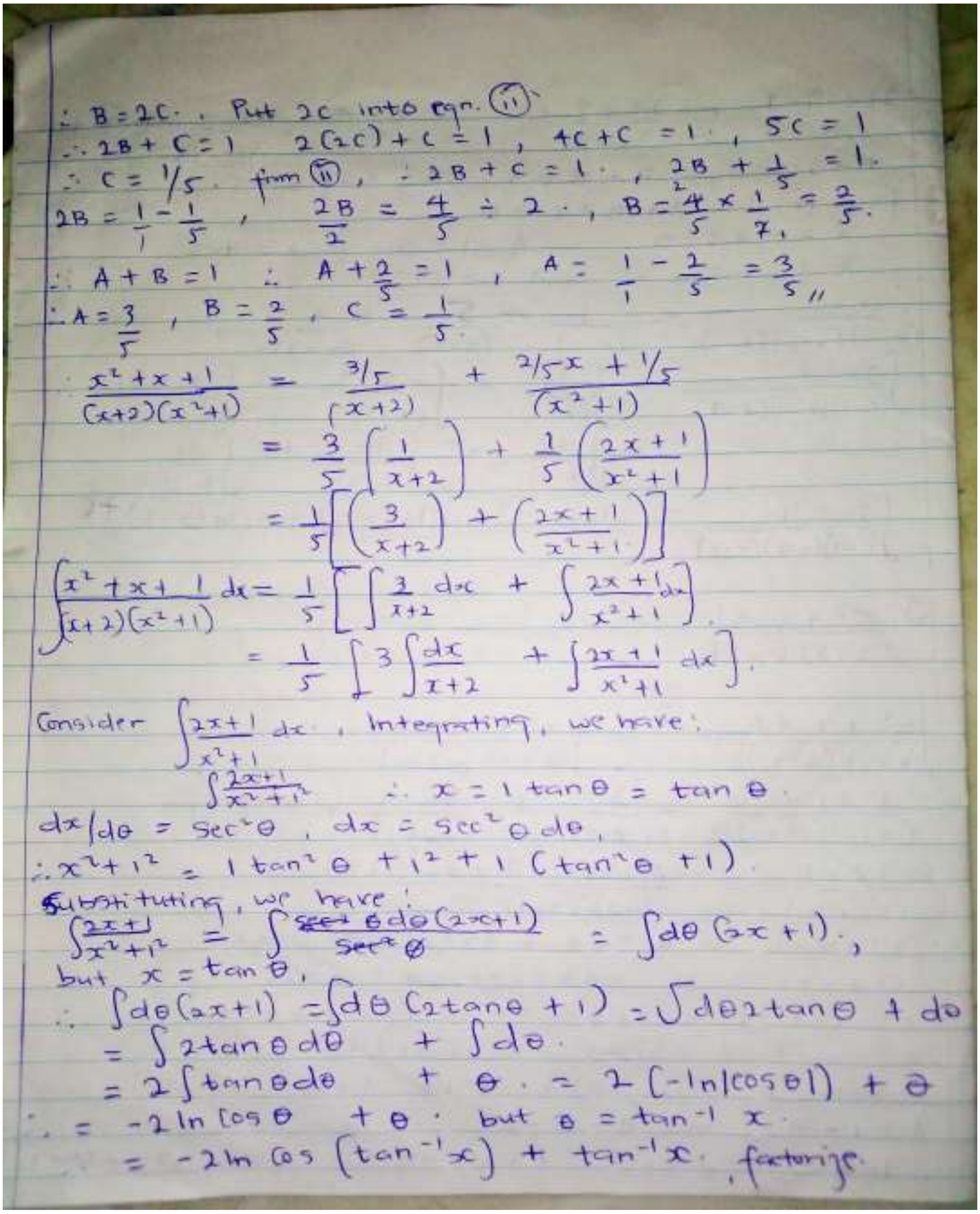

$\therefore B = 2C$. Put 2C into eqn. (ii)

$\therefore 2B + C = 1 \quad 2(2C) + C = 1, \quad 4C + C = 1, \quad 5C = 1$

$\therefore C = 1/5$. from (ii), $\therefore 2B + C = 1$, $\underset{2}{2B} + \frac{1}{5} = 1$.

$2B = \frac{1}{1} - \frac{1}{5}, \quad \frac{2B}{2} = \frac{4}{5} \div 2, \quad B = \frac{4}{5} \times \frac{1}{7} = \frac{2}{5}$.

$\therefore A + B = 1 \quad \therefore A + \frac{2}{5} = 1, \quad A = \frac{1}{1} - \frac{2}{5} = \frac{3}{5}$ //

$\therefore A = \frac{3}{5}, \quad B = \frac{2}{5}, \quad C = \frac{1}{5}$.

$$\therefore \frac{x^2 + x + 1}{(x+2)(x^2+1)} = \frac{3/5}{(x+2)} + \frac{2/5x + 1/5}{(x^2+1)}$$

$$= \frac{3}{5}\left(\frac{1}{x+2}\right) + \frac{1}{5}\left(\frac{2x+1}{x^2+1}\right)$$

$$= \frac{1}{5}\left[\left(\frac{3}{x+2}\right) + \left(\frac{2x+1}{x^2+1}\right)\right]$$

$$\int \frac{x^2 + x + 1}{(x+2)(x^2+1)} dx = \frac{1}{5}\left[\int \frac{3}{x+2} dx + \int \frac{2x+1}{x^2+1} dx\right].$$

$$= \frac{1}{5}\left[3\int \frac{dx}{x+2} + \int \frac{2x+1}{x^2+1} dx\right].$$

Consider $\int \frac{2x+1}{x^2+1} dx$, Integrating, we have:

$\int \frac{2x+1}{x^2+1^2} \quad \therefore x = 1 \tan\theta = \tan\theta$.

$dx/d\theta = \sec^2\theta, \quad dx = \sec^2\theta \, d\theta$,

$\therefore x^2 + 1^2 = 1\tan^2\theta + 1^2 + 1(\tan^2\theta + 1)$.

Substituting, we have:

$\int \frac{2x+1}{x^2+1^2} = \int \frac{\cancel{\sec^2\theta} d\theta (2x+1)}{\cancel{\sec^2\emptyset}} = \int d\theta (2x+1)$,

but $x = \tan\theta$,

$\therefore \int d\theta (2x+1) = \int d\theta (2\tan\theta + 1) = \int d\theta 2\tan\theta + d\theta$

$= \int 2\tan\theta \, d\theta + \int d\theta$.

$= 2\int \tan\theta \, d\theta + \theta. = 2(-\ln|\cos\theta|) + \theta$

$\therefore = -2\ln\cos\theta + \theta$. but $\theta = \tan^{-1} x$.

$= -2\ln\cos(\tan^{-1}x) + \tan^{-1}x$. , factorize

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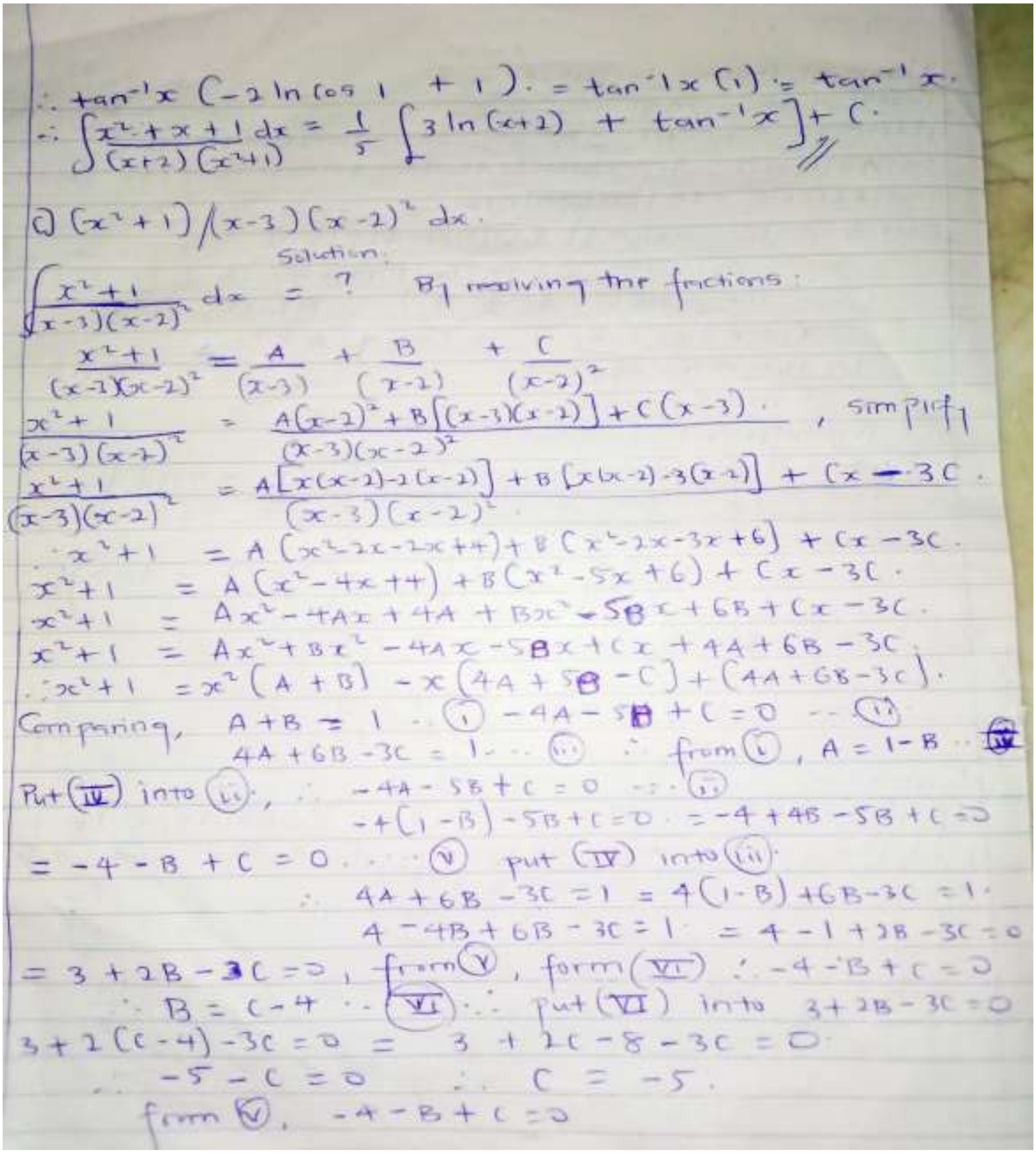

$\therefore \tan^{-1}x\,(-2\ln\cos 1 \quad + 1). = \tan^{-1}x\,(1) \therefore = \tan^{-1}x.$

$\therefore \int \frac{x^2+x+1}{(x+2)(x^2+1)}\,dx = \frac{1}{5}\left[3\ln(x+2) + \tan^{-1}x\right] + C.$ //

Q) $(x^2+1)/(x-3)(x-2)^2\,dx.$

Solution:

$\int \frac{x^2+1}{(x-3)(x-2)^2}\,dx = ?$ By resolving the fractions:

$$\frac{x^2+1}{(x-3)(x-2)^2} = \frac{A}{(x-3)} + \frac{B}{(x-2)} + \frac{C}{(x-2)^2}$$

$$\frac{x^2+1}{(x-3)(x-2)^2} = \frac{A(x-2)^2 + B[(x-3)(x-2)] + C(x-3).}{(x-3)(x-2)^2}$$, simplify

$$\frac{x^2+1}{(x-3)(x-2)^2} = \frac{A[x(x-2)-2(x-2)] + B[x(x-2)-3(x-2)] + Cx - 3C.}{(x-3)(x-2)^2}$$

$\therefore x^2+1 = A(x^2-2x-2x+4) + B(x^2-2x-3x+6] + Cx - 3C.$

$x^2+1 = A(x^2-4x+4) + B(x^2-5x+6) + Cx - 3C.$

$x^2+1 = Ax^2 - 4Ax + 4A + Bx^2 - 5Bx + 6B + Cx - 3C.$

$x^2+1 = Ax^2 + Bx^2 - 4Ax - 5Bx + Cx + 4A + 6B - 3C.$

$\therefore x^2+1 = x^2(A+B) - x(4A + 5B - C) + (4A + 6B - 3C).$

Comparing, $A + B = 1$ .. (i) $-4A - 5B + C = 0$ .. (ii)

$4A + 6B - 3C = 1$ .. (iii) $\therefore$ from (i), $A = 1 - B$ .. (IV)

Put (IV) into (ii), $\therefore -4A - 5B + C = 0$ .. (ii)

$-4(1-B) - 5B + C = 0. = -4 + 4B - 5B + C = 0$

$= -4 - B + C = 0$ .. (V) put (IV) into (iii).

$\therefore 4A + 6B - 3C = 1 = 4(1-B) + 6B - 3C = 1.$

$4 - 4B + 6B - 3C = 1 = 4 - 1 + 2B - 3C = 0$

$= 3 + 2B - 3C = 0$, from (V), form (VI) $\therefore -4 - B + C = 0$

$\therefore B = C - 4$ .. (VI) $\therefore$ put (VI) into $3 + 2B - 3C = 0$

$3 + 2(C-4) - 3C = 0 = 3 + 2C - 8 - 3C = 0$

$\therefore -5 - C = 0 \quad \therefore C = -5.$

from (V), $-4 - B + C = 0$

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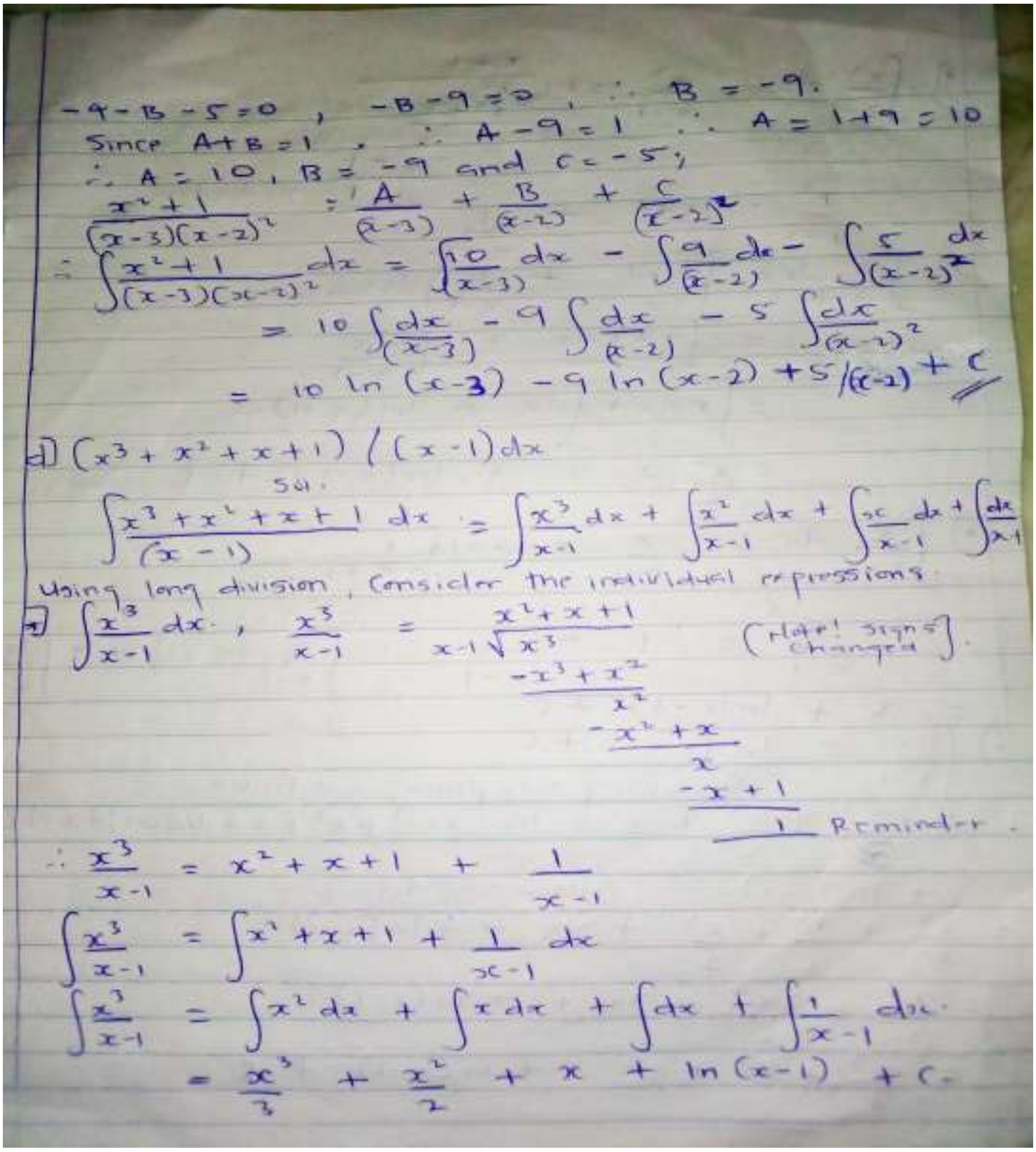

$-9 - B - 5 = 0 \quad , \quad -B - 9 = 0 \quad , \quad \therefore B = -9.$

Since $A + B = 1$, $\therefore A - 9 = 1$ $\therefore A = 1 + 9 = 10$

$\therefore A = 10$, $B = -9$ and $C = -5$;

$$\frac{x^2+1}{(x-3)(x-2)^2} = \frac{A}{(x-3)} + \frac{B}{(x-2)} + \frac{C}{(x-2)^2}$$

$$\therefore \int \frac{x^2+1}{(x-3)(x-2)^2}\,dx = \int \frac{10}{(x-3)}\,dx - \int \frac{9}{(x-2)}\,dx - \int \frac{5}{(x-2)^2}\,dx$$

$$= 10\int \frac{dx}{(x-3)} - 9\int \frac{dx}{(x-2)} - 5\int \frac{dx}{(x-2)^2}$$

$$= 10\ln(x-3) - 9\ln(x-2) + 5/(x-2) + C$$

d] $(x^3 + x^2 + x + 1) / (x-1)\,dx$

Sol.

$$\int \frac{x^3+x^2+x+1}{(x-1)}\,dx = \int \frac{x^3}{x-1}\,dx + \int \frac{x^2}{x-1}\,dx + \int \frac{x}{x-1}\,dx + \int \frac{dx}{x-1}$$

Using long division, consider the individual expressions.

i] $\int \frac{x^3}{x-1}\,dx$, $\frac{x^3}{x-1} =$

$$\begin{array}{r} x^2 + x + 1 \\ x-1 \overline{\smash{)}\, x^3 \qquad} \\ \underline{-x^3 + x^2} \\ x^2 \\ \underline{-x^2 + x} \\ x \\ \underline{-x + 1} \\ \underline{1} \end{array}$$

(Note! signs changed]. Reminder.

$$\therefore \frac{x^3}{x-1} = x^2 + x + 1 + \frac{1}{x-1}$$

$$\int \frac{x^3}{x-1} = \int x^2 + x + 1 + \frac{1}{x-1}\,dx$$

$$\int \frac{x^3}{x-1} = \int x^2\,dx + \int x\,dx + \int dx + \int \frac{1}{x-1}\,dx$$

$$= \frac{x^3}{3} + \frac{x^2}{2} + x + \ln(x-1) + C$$

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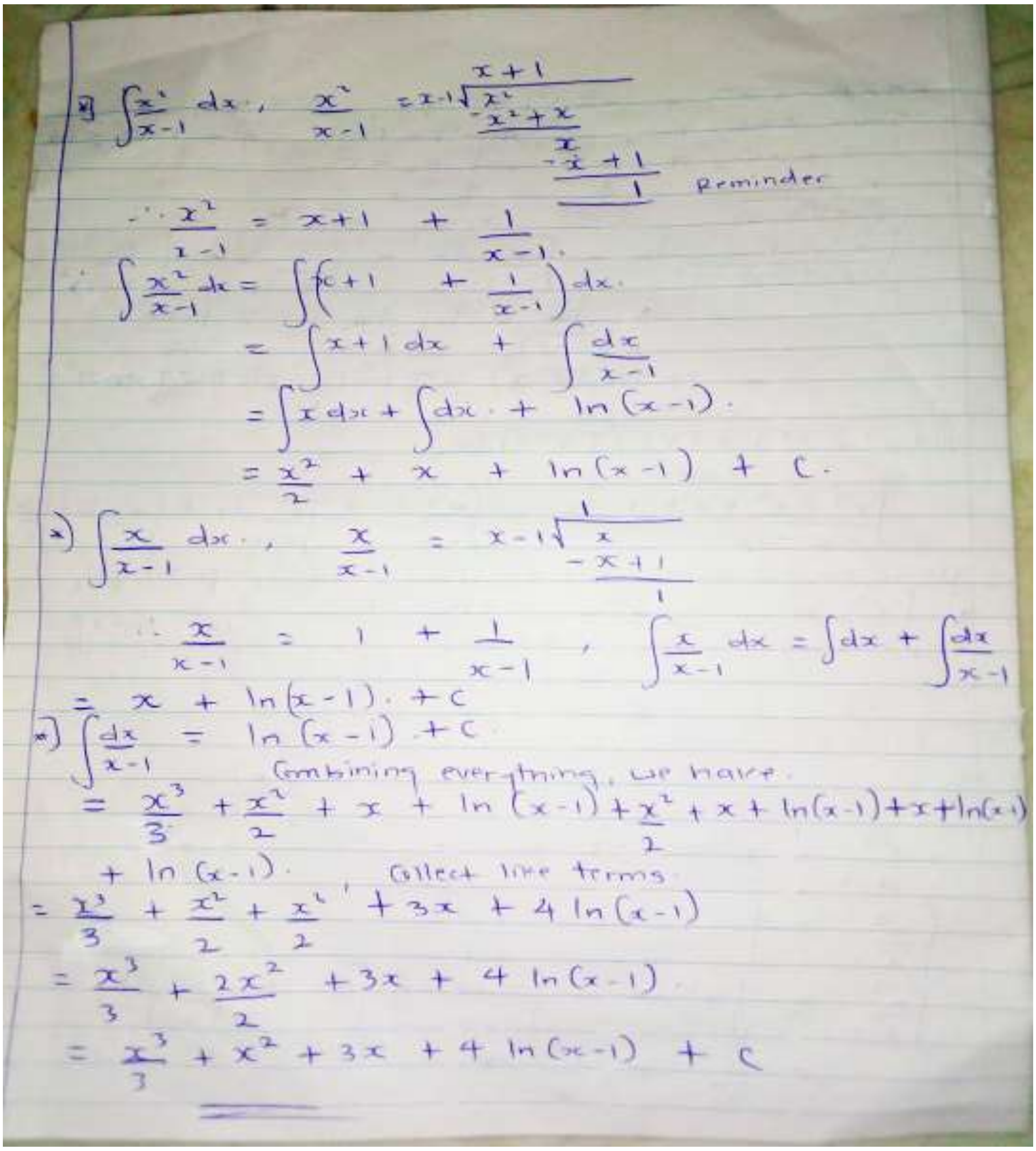

*] $\int \frac{x^2}{x-1} dx$, $\frac{x^2}{x-1}$ $= x-1 \overline{\smash{)}\, x^2}$ quotient $x+1$

$-x^2+x$

x

$-x+1$

1 Reminder

$\therefore \frac{x^2}{x-1} = x+1 + \frac{1}{x-1}$.

$\therefore \int \frac{x^2}{x-1} dx = \int \left(x+1 + \frac{1}{x-1}\right) dx$.

$= \int x+1\, dx + \int \frac{dx}{x-1}$

$= \int x\, dx + \int dx + \ln(x-1)$.

$= \frac{x^2}{2} + x + \ln(x-1) + C$.

*) $\int \frac{x}{x-1} dx$, $\frac{x}{x-1} = x-1 \overline{\smash{)}\, x}$ quotient 1

$-x+1$

1

$\therefore \frac{x}{x-1} = 1 + \frac{1}{x-1}$, $\int \frac{x}{x-1} dx = \int dx + \int \frac{dx}{x-1}$

$= x + \ln(x-1) + C$

*) $\int \frac{dx}{x-1} = \ln(x-1) + C$

Combining everything, we have.

$= \frac{x^3}{3} + \frac{x^2}{2} + x + \ln(x-1) + \frac{x^2}{2} + x + \ln(x-1) + x + \ln(x-1)$

$+ \ln(x-1)$. Collect like terms

$= \frac{x^3}{3} + \frac{x^2}{2} + \frac{x^2}{2} + 3x + 4\ln(x-1)$

$= \frac{x^3}{3} + \frac{2x^2}{2} + 3x + 4\ln(x-1)$

$= \frac{x^3}{3} + x^2 + 3x + 4\ln(x-1) + C$