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MAT104

Mechatronics ~~1001~~ Engineering

$$(1) \int \frac{(3x-1)}{(x-1)(x-2)(x-3)} dx$$

$$\frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$$

$$3x-1 = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)$$

When $x=2$

$$3(2)-1 = B(2-1)(2-3)$$

$$6-1 = B(1)(-1)$$

$$5 = -B$$

$$B = -5$$

When $x=3$

$$3(3)-1 = C(2)(1)$$

$$8 = 2C$$

$$C = 4$$

When $x=1$

$$3(1)-1 = A(-1)(-2)$$

$$2 = 2A$$

$$A = 1$$

$$\int \frac{3x-1}{(x-1)(x-2)(x-3)} dx = \int \frac{1}{x-1} dx - \int \frac{5}{x-2} dx + \int \frac{4}{x-3} dx$$

$$= \ln|x-1| - 5 \ln|x-2| + 4 \ln|x-3| + C$$

$$(2) \int \frac{x^2 + x + 1}{(x+2)(x^2+1)} dx$$

$$\frac{x^2 + x + 1}{(x+2)(x^2+1)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+1}$$

$$x^2 + x + 1 = A(x^2+1) + (Bx+C)(x+2)$$

when $x = -2$

$$4 - 2 + 1 = A(4+1)$$

$$3 = 5A$$

$$A = \frac{3}{5}$$

Expansion of the R.H.S

$$Ax^2 + A + Bx^2 + 2Bx + Cx + 2C$$

$$Ax^2 + Bx^2 + 2Bx + Cx + A + 2C$$

$$x^2(A+B) + x(2B+C) + (A+2C)$$

$$A+B = 1 \quad \text{--- (I)}$$

$$2B+C = 1 \quad \text{--- (II)}$$

$$A+2C = 1 \quad \text{--- (III)}$$

Substitute A in equ I

$$\frac{3}{5} + B = 1$$

$$B = 1 - \frac{3}{5}$$

$$B = \frac{2}{5}$$

Substitute $B =$ equ (II)

$$2\left(\frac{2}{5}\right) + c = 1$$

$$\frac{4}{5} + c = 1$$

$$c = 1 - \frac{4}{5}$$

$$c = \frac{1}{5}$$

$$\int \frac{x^2 + x + 2}{(x+2)(x^2+1)} dx = \frac{3}{5} \int \frac{dx}{x+2} + \frac{1}{5} \int \frac{2x+1}{x^2+1}$$

$$\frac{3}{5} \int \frac{dx}{x+2} + \frac{1}{5} \left[\int \frac{2x dx}{x^2+1} + \int \frac{1 dx}{x^2+1} \right]$$

$$= \frac{3}{5} \ln(x+2) + \frac{1}{5} (\ln x^2 + 1) + \frac{1}{5} (\tan^{-1} x) +$$

$$(3) \int \frac{(x^2+1)}{(x-3)(x-2)^2} dx$$

$$\frac{x^2+1}{(x-3)(x-2)^2} = \frac{A}{x-3} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$$

$$\frac{x^2+1}{(x-3)(x-2)^2} = \frac{A(x+2)^2 + B(x-3)(x-2) + C(x-3)}{(x-3)(x-2)^2}$$

$$x^2+1 = A(x+2)^2 + B(x-3)(x-2) + C(x-3)$$

$$x^2+1 = A(x^2+4x+4) + B(x^2-5x+6) + C(x-3)$$

$$x^2+1 = Ax^2 + 4Ax + 4A + Bx^2 - 5Bx + 6B + Cx - 3C$$

$$x^2+1 = Ax^2 + Bx^2 + 4Ax - 5Bx + Cx + 4A + 6B - 3C$$

$$c + x = 3$$

$$9 + 1 = 25A$$

$$A = 10/25$$

$$x^2 + 1 = (A+B)x^2 + (4A - 5B + c)x + 4A + 6B - 3c$$

$$A + B = 1 \quad \text{--- (I)}$$

$$4A - 5B + c = 0 \quad \text{--- (II)}$$

$$4A + 6B - 3c = 1 \quad \text{--- (III)}$$

Substitute A into equ (I)

$$10/25 + B = 1$$

$$B = 1 - 10/25$$

$$B = 3/5$$

Substitute A and B into equ (II)

$$\left(4 \times \frac{2}{5}\right) - \left(5 \times \frac{3}{5}\right) + c = 0$$

$$8/5 - 15/5 + c = 0$$

$$-7/5 + c = 0$$

$$c = 7/5$$

$$\int \frac{x^2 + 1}{(x-3)(x-2)^2} dx = \frac{2}{5} \int \frac{dx}{x-3} + \frac{3}{5} \int \frac{dx}{x-2} + \frac{7}{5} \int \frac{dx}{(x-2)^2}$$

$$= \frac{2}{5} \ln|x-3| + \frac{3}{5} \ln|x-2| + \frac{7}{5}(x-2)^{-1} + C$$

$$(4) \int \frac{x^3 + x^2 + x + 1}{x-1}$$

$$\cancel{2x^2} \quad x^2 + 2x + 3$$

$$x-1 \overline{) \begin{array}{r} x^3 + x^2 + x + 1 \\ -x^3 \\ \hline \end{array}}$$

$$2x^2 + x + 1$$

$$\underline{-2x^2 + 2x}$$

$$3x + 1$$

$$\underline{-3x - 3}$$

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$$\begin{array}{l} B(x^2 + x + 1) + C(x-1) \\ Bx^2 + Cx + Bx + Cx + C \\ Bx^2 + (2C+B)x + 2C \end{array}$$

$$= \int x^2 dx + \int 2x dx + \int 3x dx + \int \frac{4}{x-1} dx$$

$$= \frac{x^3}{3} + x^2 + 3x + 4 \ln|x-1| + C$$