NAME OF STUDENT: ABT OLUWAFEMISIN THADDEUS

LECTURER'S NAME: DR OYELAMI.

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DEPARTMENT: MATHEMATICS

MATRIC NO: 19/ENG05/001

Assignment

find the integral of the following:

1. $\dfrac{3x-1}{(x-1)(x-2)(x-3)}\,dx$

Solution

$$\int \frac{3x-1}{(x-1)(x-2)(x-3)}\,dx$$

$$\frac{3x-1}{(x-1)(x-2)(x-3)} \equiv \frac{A}{(x-1)} + \frac{B}{(x-2)} + \frac{C}{(x-3)}$$

$$\frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{A(x-2)(x-3)+B(x-1)(x-3)+C(x-1)(x-2)}{(x-1)(x-2)(x-3)}$$

$$3x-1 = A(x-2)(x-3)+B(x-1)(x-3)+C(x-1)(x-2)$$

when $x-1=0$

$x=1$

$$3(1)-1 = A(1-2)(1-3)+B(1-1)(1-3)+C(1-1)(1-2)$$

$$2 = A(-1)(-2)+B(0)+C(0)$$

$$2 = A(2)$$

$$2 = 2A$$

$$A = 1$$

when $x-2=0$

$x=2$

$$3(2)-1 = A(2-2)(2-3)+B(2-1)(2-3)+C(2-1)(2-2)$$

$$6-1 = A(0)+B(1)(-1)+C(0)$$

$\int \frac{3x-1}{(x-1)(x-2)(x-3)}\,dx$

- $\frac{3x-1}{(x-1)(x-2)(x-3)} \equiv \frac{A}{(x-1)} + \frac{B}{(x-2)} + \frac{C}{(x-3)}$

- $\frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{A(x-2)(x-3)+B(x-1)(x-3)+C(x-1)(x-2)}{(x-1)(x-2)(x-3)}$

$3x-1 = A(x-2)(x-3)+B(x-1)(x-3)+C(x-1)(x-2)$

when $x-1=0$

$x=1$

$3(1)-1 = A(1-2)(1-3)+B(1-1)(1-3)+C(1-1)(1-2)$

$2 = A(-1)(-2)+B(0)+C(0)$

$2 = A(2)$

$2 = 2A$

$A = 1$

when $x-2=0$

$x=2$

$3(2)-1 = A(2-2)(2-3)+B(2-1)(2-2)+C(2-1)(2-2)$

$6-1 = A(0)+B(1)(-1)+C(0)$

$5 = 0+B(-1)+C(0)$

$5 = -B$

$B = -5$

when $x-3=0$ $x=3$

$3(3)-1 = A(3-2)(3-3)+B(3-1)(3-3)+C(3-1)(3-2)$

$8 = A(0)+B(0)+C(2)(1)$

$8 = 2C$

$C = 4$

$\frac{3x-1}{(x-1)(x-2)(x-3)}\,dx = \frac{1}{x-1} + \frac{-5}{x-2} + \frac{4}{x-3}$

$\int \frac{1}{x-1} + \int \frac{-5}{x-2} + \int \frac{4}{x-3} = \ln(x-1) - 5\ln(x-2) + 4\ln(x-3)$

2. $\int \frac{x^2+x+1}{(x+2)(x^2+1)}\,dx$

SOLUTION

$$\int \frac{x^2+x+1}{(x+2)(x^2+1)}\,dx$$

$$\frac{A}{x+2} + \frac{Bx+C}{x^2+1} = \frac{x^2+x+1}{(x+2)(x^2+1)}$$

$$\frac{A(x^2+1)+(Bx+C)(x+2)}{(x+2)(x^2+1)}$$

Multiply both sides by $(x+2)(x^2+1)$

$$A(x^2+1)+(Bx+C)(x+2) = x^2+x+1$$

$$Ax^2+A+Bx^2+2Bx+Cx+2C = x^2+x+1$$

$$Ax^2+Bx^2+2Bx+Cx+2C+A = x^2+x+1$$

$$(A+B)x^2+x(2B+C)+(2C+A) = x^2+x+1$$

$$A+B = 1 \quad ---(i)$$

$$2B+C = 1 \quad ---(ii)$$

$$2C+A = 1 \quad ---(iii)$$

From eqn(i) put $B = 1-A$ in ii

$$2B+C = 1$$

$$2(1-A)+C = 1$$

$$2-2A+C = 1$$

$$-2A+C+2 = 1$$

$$-2A+C = 1-2$$

$$-2A+C = -1$$

$$C-2A = -1 \quad *2$$

$$2C+A = 1 \quad *1$$

$A+B+(B+C)+(C+A)x^2+x+1$

$A+B=1$ ---(i)

$2B+C=1$ ---(ii)

$2C+A=1$ ---(iii)

from equation put $B=1-A$ in ii

$2B+C=1$

$2(1-A)+C=1$

$2-2A+C=1$

$-2A+C+2=1$

$-2A+C=1-2$

$-2A+C=-1$

$C-2A=-1$ ×2

$2C+A=1$ ×1

$2C-4A=-2$

$2C+A=1$

$0-5A=-2-1$

$-5A=-3$

$A=3/5$

from $B=1-A$

$=1-3/5=\frac{2}{5}$

Substitute

$2C+A=1$

$2C+3/5=1$

$2C=1-3/5$

$2C=\frac{2}{5}$

$10C=2$

$C=1/5$//

$$\frac{x^2+x+1}{(x+2)(x^2+1)}\,dx = \frac{\frac{3}{5}}{x+2} + \frac{\frac{2}{5}x+\frac{1}{5}}{x^2+1}$$

$$= \frac{1}{5}\left[\frac{1}{x+2} + \frac{2x+1}{x^2+1}\right]$$

$$\frac{1}{5}\int\frac{1}{x+2} + \frac{1}{5}\int\frac{2x+1}{x^2+1}$$

3. $\int \frac{x^2+1}{(x-3)(x-2)^2}$

$$\frac{x^2+1}{(x+3)(x-2)^2} = \frac{A(x-2)^2+B(x-2)(x-3)+C(x-3)}{(x-3)(x-2)^2}$$

$$x^2+1 = A(x-2)^2+B(x-2)(x-3)+C(x-3)$$

$$x^2+1 = A(x^2-4x+4)+B(x^2-3x-2x+6)+C(x-3)$$

$$x^2+1 = A(x^2-4x+4)+B(x^2-5x+6)+C(x-3)$$

$$x^2+1 = Ax^2-4Ax+4A+Bx^2-5Bx+6B+Cx-3C$$

$$x^2+1 = Ax^2+Bx^2-4Ax-5Bx+Cx+4A+6B-3C$$

$$x^2+1 = (A+B)x^2+x(-4A-5B+C)+(4A+6B-3C)$$

$$A+B=1 \quad ---(i)$$

$$-4A-5B+C=0 \quad --(ii)$$

$$4A+6B-3C=1 \quad --(iii)$$

$$0=-4A-5B+C$$

$$-C=-4A-5B$$

$$C=4A+5B \quad ---(iv)$$

$$1=4A+6B-3C$$

$$1=4A+6B-3(4A+5B)$$

$$1=4A+6B-12A-15B$$

$\frac{x^2+1}{(x+3)(x-2)^2} = \frac{A(x-2)^2 + B(x-2)(x-3) + C(x-3)}{(x-3)(x-2)^2}$

$x^2+1 = A(x-2)^2 + B(x-2)(x-3) + C(x-3)$

$x^2+1 = A(x^2-4x+4) + B(x^2-3x-2x+6) + C(x-3)$

$x^2+1 = A(x^2-4x+4) + B(x^2-5x+6) + C(x-3)$

$x^2+1 = Ax^2 - 4Ax + 4A + Bx^2 - 5Bx + 6B + Cx - 3C$

$x^2+1 = Ax^2 + Bx^2 - 4Ax - 5Bx + Cx + 4A + 6B - 3C$

$x^2+1 = (A+B)x^2 + x(-4A-5B+C) + (4A+6B-3C)$

$A+B = 1 \quad ---(i)$

$-4A - 5B + C = 0 \quad --(ii)$

$4A + 6B - 3C = 1 \quad --(iii)$

$0 = -4A - 5B + C$

$-C = -4A - 5B$

$C = 4A + 5B \quad ---(iv)$

$12 = 4A + 6B - 3C$

$12 = 4A + 3B - 3(4A + 5B)$

$12 = 4A + 3B - 12A - 15B$

$12 = -8A - 12B \quad ---(v)$

$12 = A + B \quad *-8$

$-8 = -8A - 8B$

$-1 = -8A - 12B$

$-9 = 0 + 4B$

$-9 = 4B$

$B = -9/4$

$1 = A + B$

$1 = A - 9/4$

$A = 1 + 9/4$

$A = 13/4$

$C = 4A + 5B$

$C = 4(13/4) + 5 \cdot (-9/4)$

$C = 13 - 45/4$

$$\int \frac{x^2+1}{(x-3)(x-2)^2} = \int \frac{13}{4(x-3)} + \frac{-9}{4(x-2)} + \frac{7}{4(x-2)^2}\,dx$$

$$= \frac{13}{4}\ln(x-3) - \frac{9}{4}\ln(x-2) + \int \frac{7}{4(x-2)^2}\,dx$$

$$= \frac{7}{4}\int \frac{1}{u^2}\,dx \qquad u = x-2$$

$$\frac{du}{dx} = 1 \quad dx = 1du$$

$$\frac{7}{4}\int u^{-2}\cdot du$$

$$\frac{7}{4}\,\frac{u^{-2+1}}{-2+1} + c \qquad \frac{7}{4}\,\frac{u^{-1}}{-1} + c$$

$$\frac{-7}{4}u^{-1} + c \qquad \frac{7}{4}(x-2)^{-1} + c$$

$$= \frac{13}{4}\ln(x-3) - \frac{9}{4}\ln(x-2) + \frac{7}{4}(x-2)^{-1} + c$$

Q.7/4

Q. $\int \frac{x^3+x^2+x+1}{x-1}\,dx$

Solution

$$\int \frac{x^3+x^2+x+1}{x-1}\,dx$$

$$\int \frac{x^3+x^2+x+1}{x-1}\,dx$$

$$\begin{array}{r|l} x-1 & x^3+x^2+x+1 \\ & x^2+2x+3 \\ \hline & -x^3-x^2 \\ \hline & \quad 2x^2+x+1 \\ & \quad -2x^2-2x \\ \hline & \quad\quad 3x+1 \\ & \quad\quad -3x-3 \\ \hline & \quad\quad\quad 4 \end{array}$$

$$\frac{x^3+x^2+x+1}{x-1} = x^2+2x+3+\frac{4}{x-1}$$

$$\int x^2+2x+3+\frac{4}{x-1}\,dx$$

$$\frac{x^3}{3}+\frac{2x^2}{2}+3x+4\ln(x-1)+C$$

$$\frac{x^3}{3}+x^2+3x+4\ln(x-1)+C$$