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 MECHATRONICS ENGINEERING  
 19/ENG051019.

$$\frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{A}{(x-1)} + \frac{B}{(x+2)} + \frac{C}{(x-3)}$$

$$\frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)}{(x-1)(x-2)(x-3)}$$

Multiply through by  $(x-1)(x-2)(x-3)$

$$3x-1 = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)$$

$$3x-1 = A(x^2-5x+6) + B(x^2-4x+3) + C(x^2-3x+2)$$

$$= Ax^2 - 5Ax + 6A + Bx^2 - 4Bx + 3B + Cx^2 - 3Cx + 2C$$

$$3x-1 = (A+B+C)x^2 + (-5A-4B-3C)x + 6A+3B+2C$$

$$A+B+C = 0 \quad (1)$$

$$-5A-4B-3C = 3 \quad (2)$$

$$6A+3B+2C = -1 \quad (3)$$

from (1)

$$A = -B-C$$

Put A in 2 and 3

$$-5(-B-C) - 4B - 3C = 3$$

$$6(-B-C) + 3B + 2C = -1$$

$$5B + 5C - 4B - 3C = 3$$

$$-6B - 6C + 3B + 2C = -1$$

$$B + 2C = 3 \quad \times 3$$

$$-3B - 4C = -1 \quad \times 1$$

$$\begin{aligned} + \quad 3B + 6C &= 9 \\ -3B - 4C &= -1 \end{aligned}$$

$$2C = 8$$

$$C = 4$$

$$B + 2C = 3$$

$$B + 2(4) = 3$$

$$B + 8 = 3$$

$$B = 3 - 8$$

$$B = -5$$

$$A = -B - C$$

$$= -(-5) - 4$$

$$= 1$$

$$\int \frac{3x-1}{(x-1)(x-2)(x-3)} dx = \int \frac{1}{x-1} dx + \int \frac{-5}{x-2} dx + \int \frac{4}{x-3} dx$$

$$\ln|x-1| - 5\ln|x-2| + 4\ln|x-3| + C$$

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$$\int \frac{x^2 + x + 1}{(x+2)(x^2+1)} dx$$

$$\frac{x^2 + x + 1}{(x+2)(x^2+1)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+1}$$

$$\frac{x^2 + x + 1}{(x+2)(x^2+1)} = \frac{A(x^2+1) + (Bx+C)(x+2)}{(x+2)(x^2+1)}$$

Multiply through by  $(x+2)(x^2+1)$

$$x^2 + x + 1 = A(x^2 + 1) + (Bx + C)(x + 2)$$

$$x^2 + x + 1 = Ax^2 + A + Bx^2 + 2Bx + Cx + 2C$$

$$x^2 + x + 1 = Ax^2 + Bx^2 + 2Bx + Cx + A + 2C$$

$$x^2 + x + 1 = x^2(A + B) + x(2B + C) + A + 2C$$

$$A + B = 1 \quad (1)$$

$$2B + C = 1 \quad (2)$$

$$A + 2C = 1 \quad (3)$$

from (1)  $A = 1 - B$

Put  $1 - B$  for  $A$  and  $C$

$$(1 - B) + 2C = 1$$

$$1 - B + 2C = 1$$

$$-B + 2C = 1 - 1$$

$$-B + 2C = 0$$

$$2B + C = 1 \quad \times 1$$

$$-B + 2C = 0 \quad \times 2$$

$$2B + C = 1$$

$$-2B + 4C = 0$$

$$5C = 1$$

$$C = \frac{1}{5}$$

$$2B + C = 1$$

$$2B + \frac{1}{5} = 1$$

$$2B = 1 - \frac{1}{5}$$

$$2B = \frac{4}{5}$$

$$B = \frac{4}{5} \div 2$$

$$B = \frac{2}{5}$$

$$A = 1 - \frac{2}{5}$$

$$A = \frac{3}{5}$$

$$\int \frac{x^2 + x + 1}{(x+2)(x^2+1)} = \int \frac{\frac{3}{5}}{(x+2)} dx + \int \frac{\frac{2}{5}x + \frac{1}{5}}{x^2+1}$$

$$= \int \frac{\frac{3}{5}}{(x+2)} dx + \int \frac{\frac{2}{5}}{(x^2+1)} + \int \frac{\frac{1}{5}}{x^2+1}$$

$$= \int \frac{3}{5(x+2)} dx + \int \frac{2}{5(x^2+1)} dx + \int \frac{1}{5(x^2+1)}$$

$$\frac{3}{5} \int \frac{dx}{5(x+2)} + \frac{2}{5} \int \frac{dx}{5(x^2+1)} + \int \frac{dx}{5(x^2+1)}$$

$$= \frac{3}{5} \int \frac{dx}{5(x+2)} + \frac{2}{5} \int \frac{dx}{5(x^2+1)} + \frac{1}{5} \int \frac{dx}{5(x^2+1)}$$

$$\text{let } u = x^2 + 1$$

$$\frac{du}{dx} = 2x$$

$$du = 2x dx$$

$$x dx = \frac{du}{2}$$

$$\frac{3}{5} \int \frac{dx}{(x+2)} + \frac{2}{5} \times \frac{1}{2} \int \frac{du}{u} + \frac{1}{5} \int \frac{dx}{(x^2+1)}$$

$$\frac{3}{5} \int \frac{dx}{x+2} + \frac{1}{5} \int \frac{du}{u} + \frac{1}{5} \arctan x$$

$$\frac{3}{5} \ln(x+2) + \frac{1}{5} \ln(x^2+1) + \frac{1}{5} \arctan x + C$$

$$3 \quad \frac{x^2+1}{(x-3)(x-2)^2}$$

$$\frac{x^2+1}{(x-3)(x-2)^2} = \frac{A}{x-3} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$$

$$\frac{x^2+1}{(x-3)(x-2)^2} = \frac{A}{x-3} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$$

$$\frac{x^2+1}{(x-3)(x-2)} = \frac{A(x-2)^2 + B(x-3)(x-2) + C(x-3)}{(x-3)(x-2)}$$

Multiply through by  $(x-3)(x-2)^2$

$$x^2+1 = A(x-2)^2 + B(x-3)(x-2) + C(x-3)$$

$$x^2+1 = A(x^2-4x+4) + B(x^2-5x+6) + C(x-3)$$

$$x^2+1 = Ax^2-4Ax+4A + Bx^2-5Bx+6B + xC-3C$$

$$= Ax^2+Bx^2-4Ax-5Bx+xC+4A+6B-3C$$

$$= x^2(A+B) + x(-4A-5B+C) + (4A+6B-3C)$$

$$A+B=1 \quad (1)$$

$$-4A-5B+C=0 \quad (2)$$

$$4A+6B-3C=1 \quad (3)$$

$$A=1-B$$

Put (4) in (2) and (3)

$$-4(1-B)-5B+C=0$$

$$-4+4B-5B+C=0$$

$$-4-B+C=0$$

$$-B+C=4$$

$$4(1-B) + 6B - 3C = 1$$

$$4 - 4B + 6B - 3C = 1$$

$$4 + 2B - 3C = 1$$

$$2B - 3C = -3$$

$$2B - 3C = -3 \quad \times 1$$

$$-B + C = 4 \quad \times 2$$

$$2B - 3C = -3$$

$$-2B + 2C = 8$$

$$-C = 5$$

$$C = -5$$

$$2B - 3C = -3$$

$$2B - 3(-5) = -3$$

$$2B + 15 = -3$$

$$2B = -18$$

$$B = -9$$

$$A = 1 - B$$

$$= 1 - (-9)$$

$$= 10$$

$$\int \frac{x^2+1}{(x-3)(x-2)^2} dx = \int \frac{10}{x-3} dx + \int \frac{-9}{x-2} dx + \int \frac{-5}{(x-2)^2} dx$$

$$= \int \frac{10}{x-3} dx - \int \frac{9}{x-2} dx - \int \frac{5}{(x-2)^2} dx$$

$$10 \ln|x-3| - 9 \ln|x-2| + 5(x-2)^{-1} + C$$

$$\int \frac{x^3 + x^2 + x + 1}{x-1} dx$$

$$\begin{array}{r}
 x^2 + 2x + 3 \\
 \hline
 x-1 \overline{) \begin{array}{r} x^3 + x^2 + x + 1 \\ - (x^3 + x^2) \\ \hline 2x^2 + x + 1 \\ - (2x^2 - 2x) \\ \hline 3x + 1 \\ - (3x - 3) \\ \hline 4 \end{array} \\
 \hline
 4
 \end{array}$$

$$\int \frac{x^3 + x^2 + x + 1}{x-1} dx = \int (x^2 + 2x + 3) dx + \int \frac{4}{x-1} dx$$

$$\frac{x^3}{3} + \frac{2x^2}{2} + 3x + 4 \ln(x-1) + C$$

$$\frac{x^3}{3} + x^2 + 3x + 4 \ln(x-1) + C$$